



ARTICLE

midrangeMCP: The R package for multiple comparisons based on the Range and Midrange distributions

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Abstract

This paper introduces four new types of multiple comparison procedures. Their performance was evaluated in large experimental scenarios using Monte Carlo simulations, with a focus on comparing experimentwise error rates and statistical power against those of the traditional Tukey, Student–Newman–Keuls (SNK), and Scott–Knott tests. To facilitate the practical application of these methods, we developed the R package midrangeMCP. The proposed procedures are: the Midrange Tukey test (TM), the Midrange Student–Newman–Keuls test (SNKM), the Mean Grouping based on the Midrange (MGM), and the Mean Grouping based on the Range (MGR). The TM, MGM, and MGR tests outperformed their classical counterparts (Tukey and Scott–Knott), except under scenarios with partially true null hypotheses. The SNKM test was the only one that consistently underperformed compared to its original version (the SNK test) across all evaluated scenarios. Among the proposed methods, the MGM test stood out for its superior performance and its additional advantage of avoiding ambiguity in group assignments.

Keywords: Multiple comparisons; midrangeMCP package; R; Midrange; Range.

1. Introduction

The development of the midrangeMCP package began with the study of the externally studentized midrange distribution, which led to the creation of the SMR package (Batista & Ferreira, 2014; Batista & Ferreira, 2023). The latest update of the SMR package implemented the algorithm

developed by Bogaert (2014), improving the optimization of the Gauss-Legendre quadrature for computing the probability of the externally studentized midrange distribution.

The main goal of the `midrangeMCP` package is to implement four multiple comparison procedures based on the externally studentized range and midrange distributions, namely:

- Tukey Midrange Test (TM test) and SNK Midrange Test (SNKM test), proposed by Batista & Ferreira (2020);
- Mean Grouping based on the Studentized Midrange and Range distributions (MGM and MGR tests), proposed by Batista *et al.* (2023).

A theoretical overview of the development of these procedures is presented below. Let $Y_{(1)}, Y_{(2)}, \dots, Y_{(n)}$ be the order statistics from a random sample $Y_1, Y_2, \dots, Y_n$ of size n , drawn from a normal distribution with mean μ and variance σ^2 , that is, $Y_i \sim N(\mu, \sigma^2)$. The externally studentized *midrange* is defined as:

$$\bar{Q} = \frac{\bar{R}}{S}, \quad (1)$$

where $\bar{R} = (Y_{(n)} + Y_{(1)})/2$ is the midrange, and S is an estimator of the population standard deviation σ , with ν degrees of freedom, obtained independently of $\bar{R}$. The probability density function and cumulative distribution function of (1) were derived by Batista & Ferreira (2017) and are given by

$$f_{\bar{Q}}(\bar{q}; \mu, \sigma^2, n, \nu) = \int_0^\infty \int_{-\infty}^{x\bar{q}} 2n(n-1)x\phi(y - \mu/\sigma)\phi(2x\bar{q} - y - \mu/\sigma) \times \\ \times [\Phi(2x\bar{q} - y - \mu/\sigma) - \Phi(y - \mu/\sigma)]^{n-2} f_X(x; \nu) dy dx, \quad (2)$$

and

$$F_{\bar{Q}}(\bar{q}; \mu, \sigma^2, n, \nu) = \int_0^\infty \int_{-\infty}^{x\bar{q}} n\phi(y - \mu/\sigma)[\Phi(2x\bar{q} - y - \mu/\sigma) - \\ - \Phi(y - \mu/\sigma)]^{n-1} f_X(x; \nu) dy dx, \quad (3)$$

respectively, where the probability density function $f_X(x; \nu)$ is given by

$$f_X(x; \nu) = \frac{\nu^{\nu/2}}{\Gamma(\nu/2)2^{\nu/2-1}} x^{\nu-1} e^{-\nu x^2/2}, \quad x \geq 0. \quad (4)$$

However, $\bar{Q}$ is not an ancillary statistic with respect to μ , preventing the development of a test based directly on it. To address this, Batista & Ferreira (2014) showed that when $Y_i \sim N(0, \sigma^2)$, the probability density function and cumulative distribution function of $\bar{Q}$ simplify to:

$$f_{\bar{Q}}(\bar{q}; n, \nu) = \int_0^\infty \int_{-\infty}^{x\bar{q}} 2n(n-1)x\phi(y)\phi(2x\bar{q} - y) \times \\ \times [\Phi(2x\bar{q} - y) - \Phi(y)]^{n-2} f_X(x; \nu) dy dx, \quad (5)$$

and

$$F_{\bar{Q}}(\bar{q}; n, \nu) = \int_0^\infty \int_{-\infty}^{x\bar{q}} n\phi(y)[\Phi(2x\bar{q} - y) - \Phi(y)]^{n-1} f_X(x; \nu) dy dx. \quad (6)$$

Moreover, for $Y_i \sim N(\mu, \sigma^2)$, the expected value of $\bar{Q}$ is given by

$$E[\bar{Q}] = \frac{\mu}{\sigma} \frac{\left(\frac{\nu}{2}\right)^{1/2} \Gamma\left(\frac{\nu-1}{2}\right)}{\Gamma(\nu/2)},$$

which implies that when $\mu = 0$, we have $E[\bar{Q}] = 0$. This is a fundamental result for the development of the proposed tests in this work.

Another widely used statistic in multiple comparison procedures is the externally studentized range. Its distribution was extensively studied by Pearson (1926), Pearson (1932), Newman (1939), Pearson & Haines (1935), Pearson & Hartley (1942), Hartley (1942), Pearson & Hartley (1943), David *et al.* (1954). In the 1950s, the main MCPs based on this statistic were proposed, including the SNK test¹, the Tukey test (Tukey, 1953), and the Duncan test (Duncan, 1955).

The externally studentized range is defined as the ratio between $W = Y_{(n)} - Y_{(1)}$ and S , where S is the estimator of the population standard deviation σ , with ν degrees of freedom and independently distributed from W , that is,

$$Q = \frac{W}{S}.$$

The cumulative distribution function and probability density function of Q are given, respectively, by:

$$F_Q(q; n, \nu) = \int_0^\infty \int_{-\infty}^\infty n\Phi(y)[\Phi(xq + y) - \Phi(y)]^{n-1} f_X(x; \nu) dy dx, \quad (7)$$

and

$$f_Q(q; n, \nu) = \int_0^\infty \int_{-\infty}^\infty n(n-1)x\phi(y)\phi(xq + y)[\Phi(xq + y) - \Phi(y)]^{n-2} \\ \times f_X(x; \nu) dy dx, \quad (8)$$

where $\phi(y)$ and $\Phi(y)$ are the density and cumulative distribution functions of the standard normal distribution, and $f_X(x; \nu)$ is defined as in (4). These results are expressed in terms of the standard normal distribution, meaning that regardless of the parameters of the original normal distribution, the density and distribution functions of Q are always defined in terms of the standard normal distribution.

2. Evaluation summary of multiple comparison procedures

Two primary strategies for evaluating multiple comparison procedures (MCPs) were considered, as detailed in Batista & Ferreira (2020) and Batista *et al.* (2023). The first assesses the experiment-wise type I error rate (EER), while the second examines statistical power. Both were evaluated using Monte Carlo simulations. The results that follow are based on the studies conducted by the previously cited authors.

The performance of the TM and SNKM tests was compared against the well-established Tukey and SNK tests. Meanwhile, the MGM and MGR tests were primarily benchmarked against the

¹Initiated by Student (1927) and Newman (1939), and adapted by Keuls (1952).

Scott-Knott test. This comparison strategy reflects the methodological and conceptual rationale behind the development of each test.

Under the complete null hypothesis, the TM and SNKM tests controlled the experimentwise type I error rate either conservatively or exactly. However, in certain scenarios, the empirical EER for these tests fell notably below the nominal significance level, indicating a conservative bias. Similarly, Batista & Ferreira (2020), Carmer & Swanson (1973), and Borges & Ferreira (2003) demonstrated that the Tukey and SNK tests also maintain appropriate control over the experimentwise type I error rate. This is likewise observed for the MGM and MGR tests and the Scott-Knott test (Batista *et al.*, 2023; Silva *et al.*, 1999; Borges & Ferreira, 2003; Conrado *et al.*, 2017). However, some evaluated scenarios revealed that the Scott-Knott test occasionally exceeded the nominal significance level, albeit without substantial deviation.

Under partial alternatives, only the Tukey test consistently maintained the EER at the nominal level across a variety of experimental settings (Batista & Ferreira, 2020; Batista *et al.*, 2023; Carmer & Swanson, 1973; Borges & Ferreira, 2003). Despite this, its power has been reported to be as low as 0% in certain cases as demonstrated by Carmer & Swanson (1973), highlighting a limitation of the test.

Regarding power under the complete alternative hypothesis (H_1), the TM, MGM, MGR, Scott-Knott, and SNK tests exhibited increasing power as the number of treatments increased, with the MGM test achieving the highest gains (Batista & Ferreira, 2020; Batista *et al.*, 2023; Perecin & Malheiros, 1989). Conversely, the Tukey and SNKM tests demonstrated a decline in power under the same conditions.

In summary, selecting the most appropriate MCP strongly depends on the specific experimental context, often complicated by unknown underlying conditions. Nonetheless, the SNKM test consistently showed comparatively lower performance. The remaining procedures offer distinct advantages and limitations, highlighting the relevance of expanding the set of MCP options beyond those traditionally found in the literature.

3. Multiple Comparisons Procedures (PCMs)

Consider a random sample $Y_{11}, Y_{12}, \dots, Y_{1r}, Y_{21}, \dots, Y_{2r}, \dots, Y_{i1}, Y_{i2}, \dots, Y_{ij}, \dots, Y_{ir}, \dots, Y_{n1}, Y_{n2}, \dots, Y_{nr}$, drawn from a normal distribution $N(\mu_i, \sigma^2)$, where Y_{ij} represents the observation of the i th treatment in the j th replication, with $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, r$. The sample mean for the i th treatment is defined as

$$\bar{Y}_i = \frac{1}{r} \sum_{j=1}^r Y_{ij} = \frac{Y_{i.}}{r}. \quad (9)$$

This sample is analyzed using an analysis of variance (ANOVA) under the model

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij} = \mu_i + \epsilon_{ij}, \quad (10)$$

where $\epsilon_{ij} \sim N(0, \sigma^2)$ and $\mu_i = \mu + \tau_i$ denotes the mean effect of the i th treatment. The mean square error (MSE), an unbiased estimator of the common population variance σ^2 , is given by

$$MSE = \frac{\sum_{i=1}^n \sum_{j=1}^r (Y_{ij} - \bar{Y}_i)^2}{n(r-1)}. \quad (11)$$

It is well established by Graybill (1961) and Searle (1987), under normality, $\bar{Y}_i$ and MSE are independent, with variance estimator $\hat{V}(\bar{Y}_i) = MSE/r$.

Under the null hypothesis $H_0 : \mu_1 = \mu_2 = \dots = \mu_n = \mu$, all treatment means are equal to a common mean μ . Consequently, the ordered sample means $\bar{Y}_{(1)}, \bar{Y}_{(2)}, \dots, \bar{Y}_{(n)}$ are centered around μ . Thus, the externally studentized midrange statistic is defined as

$$\bar{Q} = \frac{\sqrt{r} [(\bar{Y}_{(1)} + \bar{Y}_{(n)})/2]}{\sqrt{MSE}}, \quad (12)$$

whose distribution depends on the unknown parameter μ under H_0 , as detailed in expressions (2) and (3). Because μ is typically unknown and rarely zero in practice, it is infeasible to use the distribution of $\bar{Q}$ directly for hypothesis testing.

To address this, we focus on the distribution of the externally studentized midrange under the specific case $\mu = 0$, given by expression (6). Since the data are centered at the unknown μ , but the distribution is centered at zero, a correction to the test statistic was necessary. As $\bar{R} = (\bar{Y}_{(1)} + \bar{Y}_{(n)})/2$, which is centered at μ , we propose the corrected statistic $\bar{R}_n = \bar{R} - \bar{Y}_\cdot^*$, where $\bar{Y}_\cdot^*$ is an estimator of μ .

Initially, we considered using the overall sample mean

$$\bar{Y}_\cdot^* = \bar{Y}_\cdot = \frac{1}{n} \sum_{i=1}^n \bar{Y}_i$$

as the estimator of μ . However, under the alternative hypothesis H_1 , this estimate converges to the average of the treatment means, $\mu = \frac{1}{n} \sum_{i=1}^n \mu_i$, where $\mu_i = E(\bar{Y}_i)$. In many cases under H_1 , $(\mu_1 + \mu_n)/2$ is close to μ , implying

$$E[\bar{R}_n] = \frac{\mu_1 + \mu_n}{2} - \mu \approx 0,$$

which results in low test power.

Under H_0 , using $\bar{Y}_\cdot$ ensures $E[\bar{R}_n] = 0$, justifying the use of the standard normal distribution for the externally studentized statistic. However, simulation studies revealed inflated experimentwise Type I error rates. It was hypothesized that this inflation arose because $\bar{R}_n$ depends on $\bar{Y}_\cdot$, introducing additional sampling variability not accounted for in the null distribution. To mitigate this, a modified test incorporating the standard error of $\bar{Y}_\cdot$ was developed. Although this controlled the Type I error rate appropriately, it suffered from reduced power.

To improve performance, we devised an estimator $\bar{Y}_\cdot^*$ that maximizes $E[\bar{R}_n]$ under H_1 while maintaining $E[\bar{R}_n] = 0$ under H_0 . This estimator corresponds to the mean of one of two groups formed by splitting the ordered means at the largest gap. Specifically, define the partition point

$$k = \arg \max_{j=1, \dots, n-1} (\bar{Y}_{(j+1)} - \bar{Y}_{(j)}).$$

If multiple maxima occur, choose

$$k = \max \{ \min(k_1, n - k_1), \min(k_2, n - k_2), \dots \}$$

among the candidate points $k_1, k_2, \dots$

The two candidate group means are then

$$\bar{Y}_1^* = \frac{\sum_{j=1}^k \bar{Y}_{(j)}}{k}$$

and

$$\bar{Y}_2^* = \frac{\sum_{j=k+1}^n \bar{Y}_{(j)}}{n-k}.$$

The estimator $\bar{Y}_\cdot^*$ is chosen as the mean of the larger group:

$$\bar{Y}_\cdot^* = \begin{cases} \bar{Y}_1^* & \text{if } k \geq n-k, \\ \bar{Y}_2^* & \text{otherwise.} \end{cases}$$

Hence, the corrected test statistic is

$$\bar{R}_n = \frac{\bar{Y}_{(1)} + \bar{Y}_{(n)}}{2} - \bar{Y}_\cdot^*, \quad (13)$$

and the minimum significant difference (MSD) was initially defined as

$$\Delta_n = \bar{q}_{(\alpha/2; n, \nu)} \sqrt{\frac{MSE}{r}} + \frac{1}{\sqrt{2n}} \sqrt{\frac{MSE}{r}}, \quad (14)$$

where $\bar{q}_{(\alpha/2; n, \nu)}$ is the 100 α /2% upper tail quantile of the distribution of $\bar{Q}$, expression (6), with n treatments and ν degrees of freedom.

3.1 Tukey test based on the midrange (TM test)

Considering the justifications presented previously, the steps for the application of the new test denoted by TM test are:

- 1) The treatment means are ordered: $\bar{Y}_{(1)}, \bar{Y}_{(2)}, \dots, \bar{Y}_{(n)}$;
- 2) The MSD is obtained by:

$$\Delta_n = \bar{q}_{(\alpha/2; n, \nu)} \sqrt{\frac{MSE}{r}} + \underbrace{\frac{1}{\sqrt{2n}} \sqrt{\frac{MSE}{r}}}_{\text{It represents the variation of } \bar{Y}_\cdot^*};$$

- 3) The value of the statistic is calculated by determining k and $\bar{Y}_\cdot^*$ as described above, by:

$$\bar{r}_n = \frac{\bar{Y}_{(1)} + \bar{Y}_{(n)}}{2} - \bar{Y}_\cdot^*;$$

- 4) If $|\bar{r}_n| \leq \Delta_n$, then the n means will be considered not different. Otherwise, go to step 5, considering $m = n$ means;
- 5) Update m by $m = m - 1$;
- 6) Considering the groups of means: $\bar{Y}_{(1)}, \bar{Y}_{(2)}, \dots, \bar{Y}_{(m)}$; $\bar{Y}_{(2)}, \bar{Y}_{(3)}, \dots, \bar{Y}_{(m+1)}$; $\dots$; $\bar{Y}_{(n-m+2)}, \dots, \bar{Y}_{(n)}$; the group of groups is given by $l = n - m + 1$;
- 7) For each one of the groups with m means, the statistic is computed by

$$\bar{r}_m = \frac{\min_j \{ \bar{Y}_{(j)} \} + \max_j \{ \bar{Y}_{(j)} \}}{2} - \bar{Y}_\cdot^*,$$

where $\bar{Y}_\cdot^*$ is obtained in the same way, as described for the set of all the n means, considering $m < n$, in this case.

- 8) For each group obtained and marked as divisible, consider m the number of means of the related group, and it should be used as MSD given by

$$\Delta_n^* = \bar{q}(\alpha/2; n, \nu) \sqrt{\frac{QME}{r}}; \quad (15)$$

- 9) If $|\bar{r}_m| \leq \Delta_n^*$, then the m means do not differ statistically; otherwise they are different at this stage;
 10) The process is repeated for all l groups of m means, redoing the steps 7, 8 and 9. After a comparison of all l groups of m means, we return to step 5, updating the value of m . This must be repeated while $m \geq 2$.

3.2 SNK test based on midrange (SNKM test)

The algorithm for the SNKM test is the same used for the TM test. The difference is shown in step 8, which follows:

8. The value of the MSD is given by

$$\Delta_m = \bar{q}(\alpha/2; m, \nu) \sqrt{\frac{QME}{r}}. \quad (16)$$

Thus, Δ_m changed as the number of means m modifies for each of the $l = n - m + 1$ groups since the argument n is changed by m , which is one of the parameters of the distribution of Q .

3.3 Mean Grouping Test Based on the Midrange (MGM Test)

The proposed multiple comparison procedure (MCP) is based on forming a partition of m ordered means at the position k , where

$$\max_j \{ \bar{Y}_{(j+1)} - \bar{Y}_{(j)} \} = \bar{Y}_{(k+1)} - \bar{Y}_{(k)}.$$

for $j = 1, 2, \dots, m-1$, and m denotes the number of means in the current group. Initially, $m = n$. In the case of ties, where multiple values of j achieve the maximum difference, say $k_1, k_2, \dots$, we define $k = \max\{\min(k_1, m - k_1), \min(k_2, m - k_2), \dots\}$, ensuring a balanced partition.

Next, we define the group means:

$$\bar{Y}_1^* = \frac{\sum_{j=1}^k \bar{Y}_{(j)}}{k}, \quad \bar{Y}_2^* = \frac{\sum_{j=k+1}^m \bar{Y}_{(j)}}{m - k}.$$

Then, we define $\bar{Y}_m^* = \bar{Y}_1^*$ if $k \geq m - k$, or $\bar{Y}_m^* = \bar{Y}_2^*$ otherwise. The steps of the MGM test are as follows:

- 1) Set $m = n$ and order the treatment means: $\bar{Y}_{(1)}, \bar{Y}_{(2)}, \dots, \bar{Y}_{(m)}$;
- 2) Identify the position k and compute $\bar{Y}_m^*$ as described above;
- 3) Compute the test statistic:

$$\bar{R}_m = \frac{\bar{Y}_{(1)} + \bar{Y}_{(m)}}{2} - \bar{Y}_m^*;$$

- 4) Compute the minimum significant difference (MSD) as:

$$\Delta_m = \bar{q}(\alpha/2; m, \nu) \sqrt{\frac{MSE}{r}} + \underbrace{\frac{1}{\sqrt{2m}} \sqrt{\frac{MSE}{r}}}_{\text{Variation of } \bar{Y}_m^*},$$

where $\bar{q}_{(\alpha/2;m,\nu)}$ is the upper $(100\alpha/2)\%$ quantile of the distribution of $\bar{Q}$ (see Expression 6), with m treatments and ν degrees of freedom;

- 5) If $|\bar{R}_m| \leq \Delta_m$, the group is considered homogeneous and marked as not partitionable. Otherwise, divide the means into two subgroups: $\bar{Y}_{(1)}, \dots, \bar{Y}_{(k)}$ and $\bar{Y}_{(k+1)}, \dots, \bar{Y}_{(m)}$, and proceed to Step 6;
- 6) For each subgroup marked as partitionable, repeat Steps 2 through 5 with the current number of means m . However, in Step 4, use the revised MSD:

$$\Delta_m = \bar{q}_{(\alpha/2;m,\nu)} \sqrt{\frac{MSE}{r}},$$

omitting the variation term of $\bar{Y}_m^*$. This adjustment increases the power of the test while maintaining the type I error rate at the nominal level, as confirmed by simulation studies. The recursive partitioning continues until no further groups can be split or until all groups consist of a single mean.

3.4 Mean Grouping Test Based on the Studentized Range (MGR Test)

A variant of the Scott–Knott test, based on the studentized range, is also proposed. The core idea of this test is analogous to that of the MGM test. To identify a suitable partition among groups, the algorithm searches for the point of maximum separation between ordered means. That is, given the ordered means $\bar{Y}_{(1)}, \bar{Y}_{(2)}, \dots, \bar{Y}_{(m)}$, the partition is made at position k satisfying

$$\max_j \{ \bar{Y}_{(j+1)} - \bar{Y}_{(j)} \} = \bar{Y}_{(k+1)} - \bar{Y}_{(k)},$$

for $j = 1, 2, \dots, m-1$. The test employs the upper $100\alpha\%$ quantile $q_{(\alpha;m,\nu)}$ of the externally studentized range distribution.

The steps of the MGR test are as follows:

- 1) Set $m = n$ and consider the m ordered treatment means: $\bar{Y}_{(1)}, \bar{Y}_{(2)}, \dots, \bar{Y}_{(m)}$;
- 2) Identify k as discussed above, based on the location of the largest gap between adjacent ordered means;
- 3) Compute the test statistic:

$$q_m = \bar{Y}_{(m)} - \bar{Y}_{(1)};$$

- 4) Compute the minimum significant difference (MSD) as

$$\Delta_m = q_{(\alpha;m,\nu)} \sqrt{\frac{MSE}{r}};$$

- 5) If $q_m \leq \Delta_m$, the m means are considered statistically indistinguishable, and the group is labeled as non-partitionable. Proceed to Step 6. Otherwise, consider the subgroup $\bar{Y}_{(1)}, \dots, \bar{Y}_{(k)}$ to be different from $\bar{Y}_{(k+1)}, \dots, \bar{Y}_{(m)}$ and proceed to Step 6;
- 6) For each group identified as partitionable, set m to the number of means in that group. Repeat Steps 2 through 5. This recursive procedure continues until no additional partitions can be made or all groups consist of a single mean.

4. The midrangeMCP package

The midrangeMCP package provides three core functions: `MRtest()`, `MRbarplot()`, and `MRwrite()`. The primary function, `MRtest()`, implements four multiple comparison procedures: MGM, MGR, SNKM, and TM. A key advantage of this function lies in its flexibility. It accepts either a numeric vector of response values, or a vector of treatment means, as well as an object containing a fitted model. Currently, supported model-fitting functions include `aov()` and `lm()`.

One notable feature of `MRtest()` is the ability to apply all four tests simultaneously in a single call, streamlining the comparison process.

The `MRbarplot()` function generates bar plots to visually display the results, clearly indicating the grouping of means according to the selected test. Finally, the `MRwrite()` function facilitates exporting the test results to external files. Supported output formats include *.csv*, *.txt*, and *.xlsx*, as well as LaTeX code. When exporting to LaTeX, the corresponding code is printed directly to the R console for easy integration into reports or manuscripts.

4.1 Dependencies

The midrangeMCP package depends on the following R packages: SMR (Batista & Ferreira, 2023), xtable (Dahl *et al.*, 2019), and WriteXLS (Schwartz, 2024). The SMR package is essential for computing quantiles from the Externally Studentized Midrange distribution and is utilized within the `MRtest()` function. The xtable and WriteXLS packages are employed in the `MRwrite()` function to facilitate the export of results in LaTeX format and as *.xlsx* files, respectively. Finally, the tcltk package (a native R package) supported the development of the graphical user interface (GUI) of the midrangeMCP package.

4.2 Installation

To install the midrangeMCP package, enter the following commands in the R console:

```
install.packages("midrangeMCP") # Install the package
library(midrangeMCP)           # Load the package
```

The following sections provide a detailed overview of the three core functions available in the midrangeMCP package.

4.3 MRtest function

The syntax of the `MRtest()` function is:

```
MRtest(y, trt = NULL, dferror = NULL, mserror = NULL,
       replication = NULL, alpha = 0.05, main = NULL,
       MCP = "all", ismean = FALSE),
```

where the arguments are:

- `y`: a vector containing the response variable, a fitted model object, or a vector of treatment means;
- `trt`: a factor vector representing the treatment levels. If `y` is a fitted model object from `lm()` or `aov()`, then `trt` specifies the name of the model component corresponding to the treatment effect. For example, if `y <- aov(response ~ treat)`, then use `trt = "treat"`. The default is `trt = NULL`, which acts as a safeguard if the user omits this argument;
- `dferror`: degrees of freedom associated with the error. This argument is required only when `y` is a vector of responses or treatment means. The default is `dferror = NULL`;

- **mserror**: mean square error. This argument is required only when *y* is a vector of responses or treatment means. The default is `mserror = NULL`;
- **alpha**: significance level for the tests; default is `alpha = 0.05`;
- **replication**: number of replicates per treatment. For unbalanced data, the harmonic mean of the replications should be provided. This argument is necessary only if *y* is a vector of means or when the data are unbalanced. The default is `replication = NULL`;
- **main**: title of the analysis;
- **MCP**: specifies which multiple comparison procedures to perform. Options include "MGM", "MGR", "SNKM", "TM", and "all" (the default), which runs all four tests. Multiple tests can be specified as a vector, e.g., `MCP = c("MGM", "TM")`;
- **ismean**: logical indicating whether *y* is a vector of treatment means (TRUE) or not (FALSE, the default).

The `MRtest()` function prints the analysis results by default, regardless of whether the output is assigned to an object. When the results are stored in an object, they are organized into four components:

- **Summary**: descriptive statistics of the response variable;
- **Statistics**: key statistics used for the tests;
- **Groups**: grouping of treatment means according to each test;
- **Tests**: names of the selected multiple comparison procedures.

Creating an object to store these results facilitates subsequent use of the `MRbarplot()` and `MRwrite()` functions.

4.4 MRbarplot function

The syntax of the `MRbarplot()` function is:

```
MRbarplot(x, MCP = "all", col = heat.colors(10),
  horiz = FALSE, ...),
```

where:

- **x**: an object returned by the `MRtest()` function;
- **MCP**: specifies the multiple comparison test(s) to be plotted. The default is "all", which plots all tests included in the `MRtest` object. To plot specific tests, provide a character vector matching the desired tests. See the `MCP` argument in the `MRtest` function (Section `MRtest` function);
- **col**: specifies the colors of the bars in the plot. The default is `heat.colors(10)`, where the number 10 defines the number of colors. For a grayscale plot, use `gray.colors(10)`;
- **horiz**: logical argument determining the orientation of the bars. The default is `FALSE`, which produces vertical bars. Setting `horiz = TRUE` plots the bars horizontally.

4.5 MRwrite function

The syntax of the `MRwrite()` function is:

```
MRwrite(x, MCP = "all", extension = "csv",
  dataMR = "all")
```

where:

- **x**: an object returned by the `MRtest()` function;

- **MCP**: specifies the multiple comparison tests for which results will be exported. The default is "all", which exports results for all tests included in the `MRtest` object. To export specific tests, provide a character vector of test names. See the `MCP` argument in the `MRtest` function (Section `MRtest` function);
- **extension**: file format for export. Available options are "csv", "txt", "xlsx" and "latex". The default is "csv";
- **dataMR**: selects which results to export. Options include "groups" (exports treatment mean groups from the tests specified in `MCP`), "summary" (exports descriptive statistics of the response variable), or "all" (exports both groups and summary). The default is "all".

All files are exported to the working directory, except for the LaTeX code, which is output directly to the R console rather than saved as a file.

5. Advantages of using the midrangeMCP package

The `midrangeMCP` package includes the `guimidrangeMCP()` function, which provides a graphical user interface (GUI) that allows users to access the package's main functionalities without the need to write command-line code (see Figure 1). This function enables users to perform data analysis, generate graphical presentations, and export results with a single click.

Such an interface is particularly advantageous for users who are not familiar with R syntax or who prefer a more user-friendly environment focused on data analysis rather than programming.

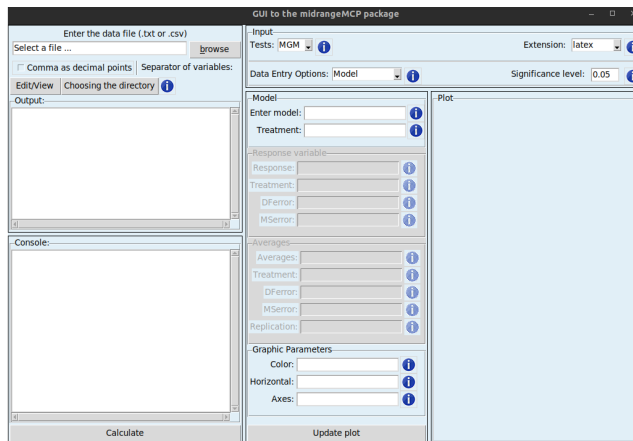


Figure 1. User Interface of the `midrangeMCP` package.

6. Applications

The `midrangeMCP` package will be demonstrated using an experiment from Steel & Torrie (1980). First, the analytical development of the tests will be discussed, followed by detailed instructions on how to use the package. A less common data input format in routine analyses is the vector of treatment means, which will also be demonstrated in this example. It will be shown that by providing only the treatment means, along with the residual mean square, degrees of freedom, and number of replications, the `MRtest()` function is able to perform all four proposed tests. The experiment will be referred to as *Example 1*.

Example 1. This experiment was conducted using a completely randomized design to evaluate the effect of bacteria on the nitrogen content (in milligrams) of red clover plants. The experimental data are presented in Table 1.

Table 1. Nitrogen content data from red clover plants related to the bacteria under study.

Type of Bacteria	Repetições				
	R_1	R_2	R_3	R_4	R_5
3DOK1	19,4	32,6	27,0	32,1	33,0
3DOK5	17,7	24,8	27,9	25,2	24,3
3DOK4	17,0	19,4	9,1	11,9	15,8
3DOK7	20,7	21,0	20,5	18,8	18,6
3DOK13	14,3	14,4	11,8	11,6	14,2
COMPOS	17,3	19,4	19,1	16,9	20,8

6.1 Using the midrangeMCP Package

The first function from the `midrangeMCP` package to be demonstrated is `MRtest()`. To begin, a file named `datast1980.txt` was created to store the dataset from *Example 1*. In R, the data can be imported using the following command:

```
# Load midrangeMCP package
library(midrangeMCP)
# Import data
dados <- read.table("datast1980.txt", h = T, dec = ",")
dados$trt <- factor(dados$trt)

attach(dados) # Attaching the data object
head(dados) # output of the first six rows
#>   trt    y
#> 1   1 19.4
#> 2   1 32.6
#> 3   1 27.0
#> 4   1 32.1
#> 5   1 33.0
#> 6   2 17.7
```

- Next, the analysis of variance (ANOVA) is performed to obtain the mean square error and the residual degrees of freedom:

```
# Anova
anava <- aov(y~trt)
```

As discussed in Subsection `MRtest` function, the `y` and `trt` arguments of the `MRtest()` function support three types of input formats:

- `y` as an object returned by the `aov()` function. In this case, the `trt` argument should specify the name of the column that contains the treatment levels:

```
MRtest(y = anava, trt = "trt")
```

```
#> MCP's based on distributions of the studentized midrange and range
#>
#> Study:  anava ~ "trt"
#>
#> Summary:
#>   Means  std r  Min  Max
#> 1 28.82 5.80 5 19.4 33.0
#> 2 23.98 3.78 5 17.7 27.9
#> 3 14.64 4.12 5  9.1 19.4
#> 4 19.92 1.13 5 18.6 21.0
#> 5 13.26 1.43 5 11.6 14.4
#> 6 18.70 1.60 5 16.9 20.8
#>
#> Mean Grouping Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange  Ext.DMS  Int.DMS
#>   19.88667 17.26515 11.78867 24 6      1.004855 1.986204 1.542946
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4
#>
#> Mean Grouping Range Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Range      DMS
#>   19.88667 17.26515 11.78867 24 6   4.372651 6.714167
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g2
#> 6 18.70      g2
#> 3 14.64      g3
#> 5 13.26      g3
#>
#> SNK Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange      DMS
#> comp1 19.8867 17.2651 11.7887 24 6      1.0049 1.9862
#> comp2 19.8867 17.2651 11.7887 24 5      1.0555 2.0640
```

```

#> comp3 19.8867 17.2651 11.7887 24 4      1.1278 2.1750
#> comp4 19.8867 17.2651 11.7887 24 3      1.2422 2.3506
#> comp5 19.8867 17.2651 11.7887 24 2      1.4594 2.6841
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98     g1g2
#> 4 19.92      g2
#> 6 18.70     g2g3
#> 3 14.64     g2g3
#> 5 13.26      g3
#>
#> Tukey Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange  Ext.DMS  Int.DMS
#> 19.88667 17.26515 11.78867 24 6      1.004855 1.986204 1.542946
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4

```

- `y` as a vector containing the response variable, and `trt` as a vector specifying the treatment levels. In this case, the values for the mean square error and the degrees of freedom of the residual must be explicitly provided through the `mserror` and `dferror` arguments of the `MRtest()` function, respectively. Thus, we have:

```

glerror <- df.residual(anava)
qmerror <- deviance(anava) / glerror
# Applying the tests
MRtest(y = y,
trt = trt,
dferror = glerror,
mserror = qmerror)

#> MCP's based on distributions of the studentized midrange and range
#> initiated
by \citet{Student1927} and \citet{Newman1939}, and adapted by \citet{Keuls1952}
#> Study: y ~ trt
#>
#> Summary:
#>   Means std r Min Max
#> 1 28.82 5.80 5 19.4 33.0
#> 2 23.98 3.78 5 17.7 27.9
#> 3 14.64 4.12 5 9.1 19.4

```

```
#> 4 19.92 1.13 5 18.6 21.0
#> 5 13.26 1.43 5 11.6 14.4
#> 6 18.70 1.60 5 16.9 20.8
#>
#> Mean Grouping Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange  Ext.DMS  Int.DMS
#>   19.88667 17.26515 11.78867 24 6      1.004855  1.986204  1.542946
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4
#>
#> Mean Grouping Range Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Range      DMS
#>   19.88667 17.26515 11.78867 24 6   4.372651  6.714167
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g2
#> 6 18.70      g2
#> 3 14.64      g3
#> 5 13.26      g3
#>
#> SNK Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange      DMS
#> comp1 19.8867 17.2651 11.7887 24 6      1.0049  1.9862
#> comp2 19.8867 17.2651 11.7887 24 5      1.0555  2.0640
#> comp3 19.8867 17.2651 11.7887 24 4      1.1278  2.1750
#> comp4 19.8867 17.2651 11.7887 24 3      1.2422  2.3506
#> comp5 19.8867 17.2651 11.7887 24 2      1.4594  2.6841
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g1g2
#> 4 19.92      g2
```

```
#> 6 18.70    g2g3
#> 3 14.64    g2g3
#> 5 13.26    g3
#>
#> Tukey Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange  Ext.DMS  Int.DMS
#>   19.88667 17.26515 11.78867 24 6      1.004855  1.986204  1.542946
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4
```

- `y` as a vector of treatment means, and `trt` as a vector specifying the corresponding treatment levels. In this case, the `MRtest()` function must be informed that the `y` argument represents means by setting `ismean = TRUE`. Additionally, the number of replications, the mean square error, and the residual degrees of freedom must be specified using the `replication`, `mserror`, and `dferror` arguments, respectively. Suppose that in *Example 1*, the available information from the experiment consists only of the analysis of variance, the treatment means, and the number of replications. In this scenario, we have:

```
# Anova
anova(anava)

#> Analysis of Variance Table
#>
#> Response: y
#>          Df Sum Sq Mean Sq F value    Pr(>F)
#> trt        5 847.05 169.409   14.37 1.485e-06 ***
#> Residuals 24 282.93  11.789
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# Treatment means
mean.trt <- c(28.82, 23.98, 14.64, 19.92, 13.26, 18.70)
# Treatments
treat <- as.factor(1:6)
# Repetitions
repet <- 5
# Degrees of freedom
dferror <- 24
# MSE
mserror <- 11.78867
```

With this information, the `MRtest()` function can perform the tests when `ismean = TRUE`. See the results below.

```
# Analysis of MRtest function - using means

MRtest(y = mean.trt,
trt = treat,
dferror,
mserror,
replication = repet,
ismean = TRUE)

#> MCP's based on distributions of the studentized midrange and range
#>
#> Study: mean.trt ~ treat
#>
#> Summary:
#> Means r
#> 1 28.82 5
#> 2 23.98 5
#> 3 14.64 5
#> 4 19.92 5
#> 5 13.26 5
#> 6 18.70 5
#>
#> Mean Grouping Midrange Test
#>
#> Statistics:
#> Exp.Mean CV MSerror Df n Stud.Midrange Ext.DMS Int.DMS
#> 19.88667 17.26515 11.78867 24 6 1.004855 1.986204 1.542946
#>
#> Groups:
#> Means Groups
#> 1 28.82 g1
#> 2 23.98 g2
#> 4 19.92 g3
#> 6 18.70 g3
#> 3 14.64 g4
#> 5 13.26 g4
#>
#> Mean Grouping Range Test
#>
#> Statistics:
#> Exp.Mean CV MSerror Df n Stud.Range DMS
#> 19.88667 17.26515 11.78867 24 6 4.372651 6.714168
#>
#> Groups:
#> Means Groups
#> 1 28.82 g1
#> 2 23.98 g2
#> 4 19.92 g2
#> 6 18.70 g2
#> 3 14.64 g3
```

```

#> 5 13.26      g3
#>
#> SNK Midrange Test
#>
#> Statistics:
#>      Exp.Mean      CV MSerror Df n Stud.Midrange      DMS
#> comp1 19.8867 17.2652 11.7887 24 6      1.0049 1.9862
#> comp2 19.8867 17.2652 11.7887 24 5      1.0555 2.0640
#> comp3 19.8867 17.2652 11.7887 24 4      1.1278 2.1750
#> comp4 19.8867 17.2652 11.7887 24 3      1.2422 2.3506
#> comp5 19.8867 17.2652 11.7887 24 2      1.4594 2.6841
#>
#> Groups:
#> Means Groups
#> 1 28.82      g1
#> 2 23.98      g1g2
#> 4 19.92      g2
#> 6 18.70      g2g3
#> 3 14.64      g2g3
#> 5 13.26      g3
#>
#> Tukey Midrange Test
#>
#> Statistics:
#>      Exp.Mean      CV MSerror Df n Stud.Midrange Ext.DMS Int.DMS
#> 19.88667 17.26515 11.78867 24 6      1.004855 1.986204 1.542946
#>
#> Groups:
#> Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4

```

However, users may not want to perform all available tests. In such cases, you can use the MCP argument to select specific tests. For example, to run the MGM and MGR tests, we use:

```

MRtest(y = anava,
trt = "trt",
MCP = c("MGM", "MGR"))

#> MCP's based on distributions of the studentized midrange and range
#>
#> Study:  anava ~ "trt"
#>
#> Summary:
#> Means std r Min Max
#> 1 28.82 5.80 5 19.4 33.0

```

```
#> 2 23.98 3.78 5 17.7 27.9
#> 3 14.64 4.12 5 9.1 19.4
#> 4 19.92 1.13 5 18.6 21.0
#> 5 13.26 1.43 5 11.6 14.4
#> 6 18.70 1.60 5 16.9 20.8
#>
#> Mean Grouping Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange  Ext.DMS  Int.DMS
#>   19.88667 17.26515 11.78867 24 6      1.004855  1.986204  1.542946
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4
#>
#> Mean Grouping Range Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Range      DMS
#>   19.88667 17.26515 11.78867 24 6    4.372651  6.714167
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g2
#> 6 18.70      g2
#> 3 14.64      g3
#> 5 13.26      g3
```

Even when the results of the `MRtest()` function are stored in an object, they are still printed to the R console. However, these results can also be accessed programmatically, as shown below.

```
teste.TM <- MRtest(y = anava,
trt = "trt",
MCP = c("TM"))

#> MCP's based on distributions of the studentized midrange and range
#>
#> Study:  anava ~ "trt"
#>
#> Summary:
#>   Means std r  Min  Max
#> 1 28.82 5.80 5 19.4 33.0
```

```

#> 2 23.98 3.78 5 17.7 27.9
#> 3 14.64 4.12 5 9.1 19.4
#> 4 19.92 1.13 5 18.6 21.0
#> 5 13.26 1.43 5 11.6 14.4
#> 6 18.70 1.60 5 16.9 20.8
#>
#> Tukey Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange  Ext.DMS  Int.DMS
#> 19.88667 17.26515 11.78867 24 6      1.004855 1.986204 1.542946
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4

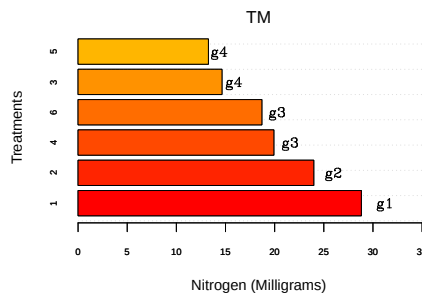
```

After creating an object with the `MRtest()` function, the other functions in the `midrangeMCP` package, namely `MRbarplot()` and `MRwrite()`, become straightforward to use. Next, we demonstrate the use of the `MRbarplot()` function with the `teste.TM` object. Observe the following code.

```

MRbarplot(teste.TM, xlab = "Nitrogen (Milligrams)",
ylab = "Treatments", horiz = TRUE)

```



If the user selects more than one test in the `MRtest()` function (via the `MCP` argument), the `MRbarplot()` function will plot the results of all chosen tests. See the example below.

```

# Default MCP = "all"
teste.all <- MRtest(y = anava, trt = "trt")

#> MCP's based on distributions of the studentized midrange and range
#>
#> Study:  anava ~ "trt"
#>
#> Summary:
#>   Means std r  Min  Max
#> 1 28.82 5.80 5 19.4 33.0

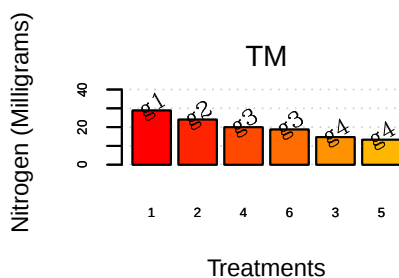
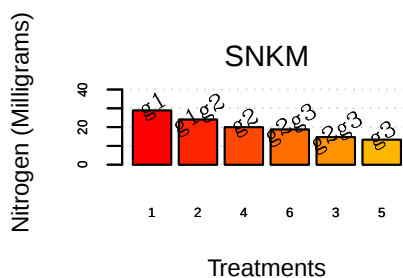
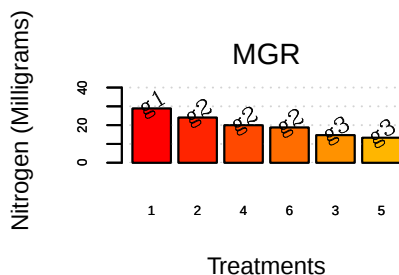
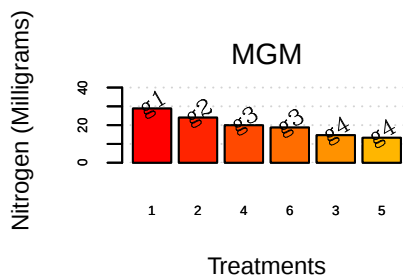
```

```
#> 2 23.98 3.78 5 17.7 27.9
#> 3 14.64 4.12 5 9.1 19.4
#> 4 19.92 1.13 5 18.6 21.0
#> 5 13.26 1.43 5 11.6 14.4
#> 6 18.70 1.60 5 16.9 20.8
#>
#> Mean Grouping Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange  Ext.DMS  Int.DMS
#>   19.88667 17.26515 11.78867 24 6      1.004855  1.986204  1.542946
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4
#>
#> Mean Grouping Range Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Range      DMS
#>   19.88667 17.26515 11.78867 24 6   4.372651  6.714167
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g2
#> 6 18.70      g2
#> 3 14.64      g3
#> 5 13.26      g3
#>
#> SNK Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange      DMS
#> comp1  19.8867 17.2651 11.7887 24 6      1.0049  1.9862
#> comp2  19.8867 17.2651 11.7887 24 5      1.0555  2.0640
#> comp3  19.8867 17.2651 11.7887 24 4      1.1278  2.1750
#> comp4  19.8867 17.2651 11.7887 24 3      1.2422  2.3506
#> comp5  19.8867 17.2651 11.7887 24 2      1.4594  2.6841
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
```

```
#> 2 23.98    g1g2
#> 4 19.92      g2
#> 6 18.70    g2g3
#> 3 14.64    g2g3
#> 5 13.26      g3
#>
#> Tukey Midrange Test
#>
#> Statistics:
#>   Exp.Mean      CV  MSerror Df n Stud.Midrange  Ext.DMS  Int.DMS
#> 19.88667 17.26515 11.78867 24 6      1.004855 1.986204 1.542946
#>
#> Groups:
#>   Means Groups
#> 1 28.82      g1
#> 2 23.98      g2
#> 4 19.92      g3
#> 6 18.70      g3
#> 3 14.64      g4
#> 5 13.26      g4
```

```
# Plotting the results
```

```
MRbarplot(teste.all, xlab = "Treatments",
ylab = "Nitrogen (Milligrams)")
```



7. Conclusion

The midrangeMCP package offers several valuable features. Firstly, it enables users with limited knowledge of R to perform data analyses easily through a graphical user interface (GUI), as illustrated in Figure 1. The package provides diverse output options, including LaTeX code generated

directly in the console to present results in well-formatted tables. Additionally, the package allows users to obtain analysis results even without direct access to the raw dataset, by simply providing some experimental characteristics.

As a future enhancement, the development of a web-based version is planned. This would allow users to access the package via web browsers on personal computers or mobile devices, thereby increasing the accessibility and convenience of the analyses.

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Conflicts of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization: BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Data curation:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Formal analysis:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Funding acquisition:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Investigation:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Methodology:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Project administration:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Software:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Resources:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Supervision:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Validation:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Visualization:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Writing – original draft:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P. **Writing – review and editing:** BATISTA, B. D. O.; FERREIRA, D. F.; ALVES, H. J. P.

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