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A bayesian mixed Beta model applied to multivariate sensory data

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Abstract

Sensory evaluation studies often rely on hedonic scales that yield ordinal and bounded data. Traditional statistical techniques, although widely applied, often fail to account to the boundedness and interdependencies characteristic of sensory responses. This study proposes a Bayesian beta regression framework specifically designed for sensory data, aiming to extend existing methodologies by addressing the joint behavior of multiple attributes. The approach models scores rescaled to the $(0,1)$ interval via a beta distribution, effectively capturing formulation effects and correlations among sensory attributes. To this end, we assumed a hierarchical structure for the regression coefficients. We chose weak prior distributions to avoid strong or subjective assumptions that might distort the results thereby, making the analysis more robust. Without strong restrictions on the prior model, the posterior inference was conducted via Markov chain Monte Carlo (MCMC) methods. By jointly analyzing all attributes within a hierarchical structure, the method enables direct estimation of inter-attribute associations and offers a more integrated interpretation of product performance. Applied to a grape juice acceptance study, the model not only identified the most preferred formulations but also unveiled meaningful patterns across sensory dimensions. With this Bayesian construction, we present a singular contribution that provides a reliable alternative for researchers seeking to analyze bounded sensory responses while simultaneously exploring the multivariate nature of consumer perception.

Keywords: Hierarchical Models; Correlation; Random Effects; Highest Density Intervals (HDIs); Markov Chain Monte Carlo (MCMC); Conjoint Analysis.

1. Introduction

Consumer perception and sensory acceptance are key determinants of a product's success in the food industry, making sensory analysis an essential tool for guiding product development, quality control, and market alignment (Huang, 2024). Since the early 2000s, the field has evolved from

traditional taste assessment to incorporate multidimensional constructs such as emotional response, consumption context, and perceived well-being (Moskowitz, 2008). This progression mirrors historical advancements, spanning 19th-century psychophysics to the development of the 9-point hedonic scale and its modern alternatives (Meiselman *et al.*, 2022). Nevertheless, the inherent complexity of sensory data, characterized by ordinality, bounded intervals, and panelist-specific variability, necessitates statistical frameworks that preserve data structure while ensuring inferential validity.

Conventional methodologies, including ANOVA-based analysis in linear models, frequently disregard the bounded and ordinal nature of sensory data by treating discrete scores as continuous variables (Christensen & Brockhoff, 2013). Although cumulative logit models offer a theoretically sound alternative for ordinal responses (Agresti, 2012; McCullagh & Nelder, 1989), their practical utility is hampered by overparameterization, convergence instability, and the need for category grouping (Ugba *et al.*, 2021; Gadrach *et al.*, 2022). Furthermore, independent analysis of sensory attributes neglects inter-attribute correlations, while asymmetric distributions—common in hedonic scaling—may induce spurious dependencies (Bardsley, 2014). Such limitations underscore the need for a unified framework capable of joint modeling under bounded, multivariate constraints.

Over the past two decades, methodological developments have increasingly explored beta regression as a powerful tool for modeling bounded continuous data. Since its seminal introduction by Ferrari & Cribari-Neto (2004), this class of models has undergone numerous extensions. Within the Bayesian framework, several strategies have been proposed, including: (i) beta regression for conditional mean estimation (Branscum *et al.*, 2007), (ii) beta-rectangular mixtures for greater robustness to outliers (Bayes *et al.*, 2012), and (iii) nonparametric Bayesian approaches for estimating the entire response distribution (Barrientos *et al.*, 2017). Although these methods offer considerable flexibility, most were not explicitly designed to address the hierarchical and multivariate dependencies that are characteristic of sensory data structures. In more recent contributions, Revuelta *et al.* (2022) proposed a beta factor model for doubly bounded data, and Di Brisco *et al.* (2023) developed functional beta regression to accommodate curve-valued predictors. Nonetheless, to the best of our knowledge, no studies have applied Bayesian beta regression to the joint modeling of multiple sensory attributes within a unified and hierarchical framework.

This study proposes a unified Bayesian beta regression framework that directly addresses key methodological limitations observed in previous approaches. Specifically, existing models often fail to simultaneously account for the hierarchical nature of sensory data and the multivariate dependency among multiple attributes. Our approach overcomes these gaps by modeling the bounded response variable using a Beta distribution, with its mean linked to a hierarchical linear predictor that includes both fixed effects (e.g., formulation, attribute) and crossed random effects (e.g., panelist- and attribute-specific variation). A structured covariance matrix is incorporated to capture the correlations among sensory attributes explicitly. The Bayesian formulation offers several advantages: (1) natural uncertainty quantification via posterior distributions, (2) shrinkage and stabilization of parameter estimates through hierarchical priors, which is particularly beneficial for small sample sizes, and (3) comprehensive model evaluation using predictive criteria such as leave-one-out cross-validation (LOO-CV) (Vehtari *et al.*, 2017). Additionally, the model accommodates a wide range of distributional shapes commonly observed in sensory data (e.g., skewed, U-shaped, or symmetric) and mitigates aggregation bias by including random effects (Figueroa-Zúñiga *et al.*, 2013).

The remainder of this article is organized as follows. Section 2 briefly reviews the theoretical foundations of Bayesian inference relevant to our approach. Section 3 details the model specification and estimation procedure via MCMC. Section 4 demonstrates the framework's application to real sensory data. Section 5 provides a critical discussion of the applied methodology, and Section 6 discusses the implications and outlines potential extensions.

2. Foundations of Bayesian Inference

This section presents a brief review of the Bayesian inference framework, with its theoretical foundations primarily drawn from Paulino *et al.* (2018), Bernardo & Smith (2009), and Gelman *et al.* (1995), who provide a principled approach for updating knowledge about unknown parameters and underpin the methodology adopted in this study.

Bayesian inference offers a coherent framework for updating beliefs about unknown parameters $\theta \in \Theta$ based on observed data, in contrast to the classical approach, which treats θ as a fixed but unknown constant. The Bayesian paradigm models θ as a random variable with a prior distribution $h(\theta)$ that encodes prior knowledge or expert opinion.

After observing data γ , this prior distribution is updated via Bayes' theorem to obtain the posterior distribution:

$$h(\theta | \gamma) = \frac{f(\gamma | \theta)h(\theta)}{\int_{\Theta} f(\gamma | \theta)h(\theta) d\theta}, \quad \theta \in \Theta, \quad (1)$$

where $f(\gamma | \theta)$ is the likelihood function, and the denominator ensures that the posterior integrates to one. In practice, the posterior is often expressed up to proportionality:

$$h(\theta | \gamma) \propto f(\gamma | \theta)h(\theta). \quad (2)$$

For a sample $(\gamma_1, \dots, \gamma_n)$, the expression extends naturally as:

$$h(\theta | \gamma_1, \dots, \gamma_n) \propto \left[\prod_{i=1}^n f(\gamma_i | \theta) \right] h(\theta). \quad (3)$$

The posterior distribution $h(\theta | \gamma_1, \dots, \gamma_n)$ summarizes all available information about θ , combining prior beliefs with the observed data. It serves as the foundation for estimation, prediction, and decision-making in the Bayesian framework. When closed-form solutions are intractable, numerical methods such as MCMC algorithms (Gilks *et al.*, 1995) are employed to approximate the posterior distribution. Bayesian inference was conducted in R (R Core Team, 2024) using JAGS via the rjags package (Plummer, 2025).

For comprehensive treatments of modern Bayesian methodology, readers may consult Gelman *et al.* (2013), Kruschke (2014), and Robert *et al.* (2007).

3. Bayesian Model and Inference Procedure

The proposed hierarchical Bayesian framework provides a unified approach for modeling multivariate sensory data while accounting for panelist heterogeneity and attribute dependencies. This section details the model specification, prior selection, and computational implementation.

3.1 Hierarchical Beta Regression Model

The normalized hedonic scores were analyzed using a unified Bayesian beta regression model with random effects. Given the bounded and potentially asymmetric nature of these data, the response variable y_{ijl} , representing the score from panelist i ($i = 1, \dots, b$), formulation j ($j = 1, \dots, v$), and sensory attribute l ($l = 1, \dots, L$), was assumed to follow a Beta distribution:

$$y_{ijl} \sim \text{Beta}(\mu_{ijl}\phi, (1 - \mu_{ijl})\phi), \quad (4)$$

where μ_{ijl} denotes the expected value of y_{ijl} and ϕ is a precision parameter governing the dispersion.

The conditional mean μ_{ijl} was linked to covariates through a logit-linear predictor as follows:

$$\text{logit}(\mu_{ijl}) = \eta_{ijl} = b_0 + \beta_j + \beta_l + u_i + u_l, \quad (5)$$

where b_0 represents the global intercept; β_j denotes the fixed effect of formulation j ; and β_l corresponds to the fixed effect of attribute l . The term u_i represents the random effect associated with panelist i , for which it is common (but not required) to assume that $u_i \sim \mathcal{N}(0, \sigma_p^2)$, capturing individual-specific variability. In addition, the vector of attribute-specific random effects, $\mathbf{u}_l = \{u_{[1]}, \dots, u_{[L]}\}$, is assumed to follow a multivariate normal distribution, i.e., $\mathbf{u}_l \sim \mathcal{N}_L(\mathbf{0}, \mathbf{\Sigma})$, where $\mathbf{\Sigma}$ is the $L \times L$ covariance matrix that captures potential correlations among sensory attributes.

This hierarchical structure enables the simultaneous modeling of multiple sensory attributes while accounting for panelist-level heterogeneity and inter-attribute dependencies.

3.2 Choice of priors

To ensure estimation stability without unduly influencing the results, weakly informative prior distributions were assigned as follows (Revuelta *et al.*, 2022):

$$\begin{aligned} b_0 &\sim \mathcal{N}(0, 10), \\ \beta_{[j]} &\sim \mathcal{N}(0, 10) \quad \text{for } j = 1, \dots, \nu, \\ \beta_{[l]} &\sim \mathcal{N}(0, 10) \quad \text{for } l = 1, \dots, L, \\ \sigma_p^{-2} &\sim \text{Gamma}(0.1, 10), \\ \mathbf{\Omega} = \mathbf{\Sigma}^{-1} &\sim \text{Wishart}(\mathbf{I}_L, L + 1), \\ \phi &\sim \text{Gamma}(0.1, 10), \end{aligned}$$

where $\mathbf{\Omega}$ denotes the precision matrix of the attribute-specific random effects. A Wishart prior was assigned to $\mathbf{\Omega}$ with scale matrix \mathbf{I}_L , the identity matrix of dimension L , and degrees of freedom $L+1$. This choice ensures the positive definiteness of the covariance matrix $\mathbf{\Sigma}$ while remaining weakly informative, thereby allowing the estimation of attribute correlations without imposing strong prior constraints (Chung *et al.*, 2015). Weakly informative priors offer several advantages: they provide flexibility, reduce the risk of incorporating incorrect or overly strong assumptions, and enable inference to be primarily driven by the observed data. Additionally, they improve the robustness of the model by stabilizing estimation while maintaining general applicability across a variety of scenarios (Kass & Wasserman, 1996).

Moreover, the use of weakly informative priors in conjunction with partial pooling induced by the hierarchical structure acts as an intrinsic regularization mechanism, reducing the risk of overfitting even in moderately sized samples and high dimensional multivariate settings.

3.3 Posterior Distribution

Combining the likelihood and prior specifications, the joint posterior distribution governing the model parameters can be formally expressed as follows:

$$\begin{aligned}
 p(\boldsymbol{\theta} \mid \boldsymbol{y}) &\propto \prod_{i=1}^b \prod_{j=1}^v \prod_{l=1}^L \left[\text{Beta} \left(\gamma_{ijl} \mid \mu_{ijl}\Phi, (1 - \mu_{ijl})\Phi \right) \right]^{\mathbb{I}_{ijl}} \\
 &\times \mathcal{N}(b_0 \mid 0, 10) \\
 &\times \left[\prod_{j=1}^v \mathcal{N}(\beta_{[j]} \mid 0, 10) \right] \\
 &\times \left[\prod_{l=1}^L \mathcal{N}(\beta_{[l]} \mid 0, 10) \right] \\
 &\times \left[\prod_{i=1}^b \mathcal{N}(u_{[i]} \mid 0, \sigma_p^2) \right] \times \text{Gamma}(\sigma_p^{-2} \mid 0.1, 10) \\
 &\times \mathcal{N}_L(\boldsymbol{u}_l \mid \mathbf{0}, \boldsymbol{\Sigma}) \times \text{Wishart}(\boldsymbol{\Sigma}^{-1} \mid \boldsymbol{I}_L, L + 1) \\
 &\times \text{Gamma}(\Phi \mid 0.1, 10),
 \end{aligned} \tag{6}$$

where \mathbb{I}_{ijl} denotes an indicator function that takes the value 1 if the combination (i, j, l) is observed in the Balanced Incomplete Block (BIB) design, and 0 otherwise. The vector of all model parameters is represented by $\boldsymbol{\theta} = \{b_0, \boldsymbol{\beta}_j, \boldsymbol{\beta}_l, \boldsymbol{u}_i, \boldsymbol{u}_l, \boldsymbol{\Sigma}, \sigma_p, \Phi\}$.

The Beta density function is properly normalized:

$$\text{Beta}(\gamma \mid \alpha, \beta) = \frac{\gamma^{\alpha-1}(1-\gamma)^{\beta-1}}{B(\alpha, \beta)},$$

in which $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ is the Beta function.

The corresponding log-posterior distribution, excluding additive constants, was derived by taking the natural logarithm of Equation (6):

$$\begin{aligned}
 \log p(\boldsymbol{\theta} \mid \boldsymbol{y}) &= \sum_{i=1}^b \sum_{j=1}^v \sum_{l=1}^L \mathbb{I}_{ijl} \left[(\mu_{ijl}\Phi - 1) \log \gamma_{ijl} \right. \\
 &\quad \left. + ((1 - \mu_{ijl})\Phi - 1) \log(1 - \gamma_{ijl}) - \log B(\mu_{ijl}\Phi, (1 - \mu_{ijl})\Phi) \right] \\
 &\quad - \frac{1}{2} \left(\frac{b_0^2}{10} + \sum_{j=1}^v \frac{\beta_{[j]}^2}{10} + \sum_{l=1}^L \frac{\beta_{[l]}^2}{10} \right) \\
 &\quad + (0.1 - 1) \log \Phi - \frac{\Phi}{10} \\
 &\quad - \frac{1}{2} \left(\sum_{i=1}^b \frac{u_{[i]}^2}{\sigma_p^2} + b \log \sigma_p^2 \right) \\
 &\quad + (0.1 - 1) \log \sigma_p^{-2} - \frac{\sigma_p^{-2}}{10} \\
 &\quad - \frac{1}{2} \left(\boldsymbol{u}_l^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_l + \log |\boldsymbol{\Sigma}| \right) \\
 &\quad + \frac{L+1}{2} \log |\boldsymbol{\Sigma}^{-1}| - \frac{1}{2} \text{tr}(\boldsymbol{I}_L \boldsymbol{\Sigma}^{-1}).
 \end{aligned} \tag{7}$$

3.4 Estimation and Convergence

The joint posterior distribution (Equation 6) integrates the likelihood and prior specifications and serves as the foundation for Bayesian inference. Posterior inference was conducted via MCMC methods, targeting the log-posterior distribution described in Equation 7. Four parallel chains were initialized with dispersed starting values to ensure broad exploration of the parameter space. Each chain was run for 80,000 iterations, with the first 30,000 iterations discarded as burn-in to mitigate the influence of initial values. To reduce autocorrelation among successive samples, a thinning interval of 10 was applied. Posterior summaries were obtained after burn-in and thinning.

Parameter estimation was carried out using a hybrid algorithm within JAGS, combining Gibbs sampling and Metropolis–Hastings steps. These are Monte Carlo methods for sampling from complex posterior distributions when direct sampling is not feasible, as described by Gelman *et al.* (2013). For parameters whose full conditional distributions are available in closed form—such as normal random effects and precision parameters—standard Gibbs updates were employed. For non-conjugate components, including the logit-linked mean μ_{ijl} , Metropolis–Hastings steps were used within the Gibbs sampler. This strategy ensures computational efficiency while maintaining accurate approximation of the posterior distribution.

Convergence diagnostics were performed to ensure the reliability of the MCMC samples. These included the evaluation of Gelman–Rubin statistics (\hat{R}), which measure the potential scale reduction factor by comparing within-chain and between-chain variances. Values of \hat{R} close to 1 indicate convergence to the target posterior distribution Brooks & Gelman (1998). In this analysis, all monitored parameters presented \hat{R} values below the conventional threshold of 1.05. Effective sample sizes (ESS) were also verified, with all parameters exceeding 1,000 samples, indicating sufficient precision in the posterior summaries. Additionally, trace plots were visually inspected to confirm adequate mixing and stationarity of the MCMC chains.

3.5 Inference Targets and Uncertainty Quantification

The posterior inference focused on three key quantities of interest. First, the marginal posterior distributions of μ_{ijl} , representing the expected hedonic scores, were summarized using 95% Highest Density Intervals (HDIs) to quantify uncertainty. Second, the attribute-specific variances, denoted by σ_l^2 , were obtained from the diagonal elements of the covariance matrix Σ , characterizing the variability associated with each sensory attribute. Finally, the correlation structure among attributes was assessed through the attribute correlation matrix R , derived from the standardized version of Σ , allowing the identification of underlying associations among sensory perceptions.

3.6 Predictive Assessment

The predictive performance of the model was evaluated by explicitly computing posterior predictive densities for each observation, using the sampled values of μ_{ijl} and the precision parameter ϕ , which, in the Beta distribution parameterization of Ferrari & Cribari-Neto (2004), controls the dispersion around the mean. These densities were then employed to derive pointwise log-likelihoods, which served as the basis for model validation through Pareto-smoothed importance sampling leave-one-out PSIS-LOO. The diagnostic assessment considered Pareto \hat{k} values, ensuring they remained below 0.7 to guarantee reliable approximations, and the expected log predictive density (ELPD) was estimated along with its associated standard error (Vehtari *et al.*, 2017).

4. A real data application

A sensory evaluation study was conducted in 2025 at the Sensory Analysis Laboratory of the Department of Agroindustry, Food and Nutrition, "Luiz de Queiroz" College of Agriculture (ESALQ/USP),

in Piracicaba, Brazil, to investigate consumer acceptance across eight commercial grape juice formulations. The study protocol received ethical approval from the institutional review board (Certificate 84964124.7.0000.5395), ensuring compliance with research ethics standards.

A BIB design was employed to evaluate four natural (F_{873} , F_{661} , F_{419} , F_{571}) and four processed (F_{715} , F_{179} , F_{732} , F_{318}) formulations. A total of 98 untrained panelists assessed the samples using a standardized 5-point hedonic scale for five sensory attributes: color, acidity, sweetness, flavor, and aroma. Although ABNT (2018) recommends the 9-point hedonic scale due to its greater discriminative power, the standard also permits the use of shorter formats. In this study, the 5-point scale was selected to accommodate the profile of the assessors—untrained consumers—for whom a shorter scale facilitates comprehension and consistent application of the evaluations. Each panelist evaluated four different samples, with each formulation assessed by 49 panelists and each pair of formulations appearing exactly 21 times. The use of the BIB design aimed to reduce panelist fatigue, lower experimental costs and execution time, and provide a more reliable and efficient data structure for subsequent statistical analyses.

To accommodate the assumptions of beta regression, discrete hedonic scores (1–5) were first rescaled to the unit interval by division by five. Because the Beta distribution is supported on (0, 1) and is not defined at the boundaries, boundary values at 0 or 1 were avoided by applying the Smithson–Verkuilen adjustment (Smithson & Verkuilen, 2006):

$$y^* = \frac{\gamma(n-1) + 0.5}{n},$$

where $\gamma \in [0, 1]$ denotes the rescaled score and $n = bk$ is the total number of observations. This transformation preserves the relative ordering of responses while ensuring $y^* \in (0, 1)$ and, therefore, compatibility with the Beta regression model (Ferrari & Cribari-Neto, 2004).

After rescaling the hedonic scores to the range (0, 1) and restructuring the dataset into a stacked format suitable for unified modeling, descriptive measures, including means and variances, were computed for each formulation and attribute, which are presented in Table 1.

Table 1. Means and variances of the eight grape juice formulations for five sensory attributes, obtained in an evaluation carried out at ESALQ/USP in 2025

Attribute	F_{179}	F_{318}	F_{419}	F_{571}	F_{661}	F_{715}	F_{732}	F_{873}
Mean								
Acidity	0.751	0.714	0.714	0.689	0.686	0.633	0.653	0.675
Aroma	0.759	0.620	0.775	0.755	0.718	0.637	0.604	0.743
Color	0.877	0.841	0.837	0.828	0.743	0.759	0.535	0.800
Sweetness	0.718	0.698	0.751	0.714	0.665	0.608	0.677	0.702
Flavor	0.743	0.665	0.751	0.710	0.682	0.612	0.629	0.714
Variance								
Acidity	0.041	0.062	0.057	0.064	0.058	0.066	0.071	0.055
Aroma	0.047	0.049	0.053	0.030	0.051	0.051	0.049	0.030
Color	0.028	0.040	0.039	0.035	0.072	0.040	0.059	0.038
Sweetness	0.053	0.055	0.057	0.057	0.066	0.050	0.070	0.067
Flavor	0.050	0.064	0.067	0.047	0.068	0.056	0.063	0.068

The analysis of the mean values in Table 1 indicates that formulations F_{419} , F_{179} , and F_{873} were among the highest-rated by consumers across all sensory attributes, with mean scores exceeding 0.70 in most dimensions evaluated. In contrast, formulations F_{732} and F_{715} exhibited the lowest mean scores, particularly for color, flavor, and sweetness, suggesting lower hedonic acceptance. The reported variance measures were relatively homogeneous across formulations, although slightly higher values were observed for color and flavor, indicating greater dispersion of responses in these dimensions. Overall, the estimates obtained indicate perceptible differences among the evaluated formulations, thereby justifying subsequent modeling using a multivariate approach.

Following the exploratory assessment, a hierarchical Bayesian beta regression model was fitted to the data to jointly analyze the sensory attributes while accounting for individual variability among panelists, following the methodological procedures outlined in Section 3. Convergence diagnostics indicated satisfactory mixing for the vast majority of parameters: fixed effects for attributes and formulations, as well as the random effects for panelists, exhibited \hat{R} values close to 1.000 and below the conventional threshold of 1.05. Effective sample sizes (ESS) exceeded 24,000 across all monitored quantities, indicating high Monte Carlo precision. A few covariance elements in the attribute-level matrix Σ presented slightly elevated \hat{R} values (up to ≈ 1.2), which is common in hierarchical models with correlated random effects due to slower mixing of off-diagonal components (Gelman *et al.*, 2013). These minor deviations were not systematic and occurred alongside large ESS, suggesting no practical convergence issues.

Convergence and mixing of the MCMC chains were further corroborated by visual inspection of trace plots and posterior density estimates for representative parameters, including the global intercept (b_0), formulation effects (β_{form}), and the precision parameter (ϕ) (7, Figures 2–7). The chains displayed stable oscillations around constant mean levels, with no discernible trends, and unimodal, approximately symmetric posterior densities, indicating satisfactory convergence. Similarly, the trace plots and posterior densities for the standard deviation of the panelist random effects (σ_p) and for the fixed effects associated with sensory attributes (7, Figures 8–11) were consistent with good chain mixing and stationarity, reinforcing the reliability of the posterior summaries.

Posterior means and 95% HDIs were estimated for each formulation–attribute combination, marginalizing over the random effects of panelists. This marginal inference provides population-level acceptance estimates, facilitating comparisons between formulations while accounting for individual variability. The results are visualized in Figure 1, which presents the estimated means and uncertainty intervals for each sensory attribute.

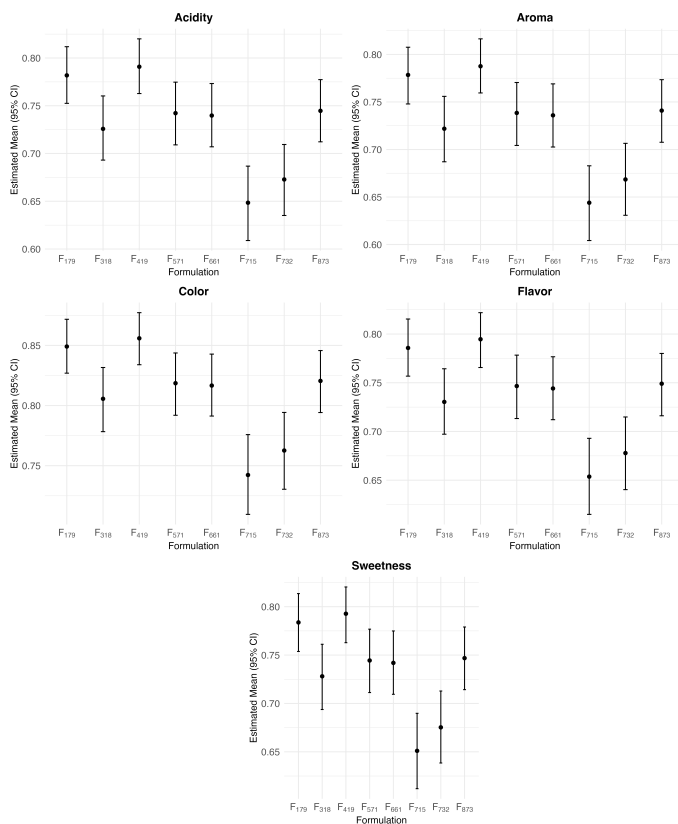


Figure 1. Estimated means and 95% HDIs of the sensory attributes for the eight grape juice formulations derived from the data collected at ESALQ/USP in 2025.

Based on the posterior means and corresponding HDIs depicted in Figure 1, pairwise comparisons were systematically performed for each sensory attribute to identify formulations with statistically significant differences. Two formulations were considered significantly different when their 95% HDIs did not overlap, indicating distinct levels of sensory acceptance. The results of these comparisons are presented in Table 2, which enumerates all formulation pairs exhibiting statistically significant differences, organized by sensory attribute.

Table 2. Pairwise comparisons between grape juice formulations that exhibited statistically significant differences in sensory acceptance for each attribute, based on non-overlapping 95% HDIs, from the study conducted at ESALQ/USP in 2025

Attribute	Formulation 1	Formulation 2	Mean 1	Mean 2
Acidity	F ₁₇₉	F ₇₁₅	0.782	0.648
	F ₁₇₉	F ₇₃₂	0.782	0.673
	F ₃₁₈	F ₄₁₉	0.726	0.791
	F ₃₁₈	F ₇₁₅	0.726	0.648
	F ₄₁₉	F ₇₁₅	0.791	0.648
	F ₄₁₉	F ₇₃₂	0.791	0.673
	F ₅₇₁	F ₇₁₅	0.742	0.648
	F ₆₆₁	F ₇₁₅	0.740	0.648
	F ₇₁₅	F ₈₇₃	0.648	0.745
Aroma	F ₇₃₂	F ₈₇₃	0.673	0.745
	F ₁₇₉	F ₇₁₅	0.778	0.644
	F ₁₇₉	F ₇₃₂	0.778	0.669
	F ₃₁₈	F ₄₁₉	0.722	0.788
	F ₃₁₈	F ₇₁₅	0.722	0.644
	F ₄₁₉	F ₇₁₅	0.788	0.644
	F ₄₁₉	F ₇₃₂	0.788	0.669
	F ₅₇₁	F ₇₁₅	0.738	0.644
	F ₆₆₁	F ₇₁₅	0.736	0.644
Color	F ₇₁₅	F ₈₇₃	0.644	0.741
	F ₇₃₂	F ₈₇₃	0.669	0.741
	F ₁₇₉	F ₇₁₅	0.849	0.742
	F ₁₇₉	F ₇₃₂	0.849	0.763
	F ₃₁₈	F ₄₁₉	0.806	0.856
	F ₃₁₈	F ₇₁₅	0.806	0.742
	F ₄₁₉	F ₇₁₅	0.856	0.742
	F ₄₁₉	F ₇₃₂	0.856	0.763
	F ₅₇₁	F ₇₁₅	0.818	0.742
Sweetness	F ₆₆₁	F ₇₁₅	0.817	0.742
	F ₇₁₅	F ₈₇₃	0.742	0.820
	F ₇₃₂	F ₈₇₃	0.763	0.820
	F ₁₇₉	F ₇₁₅	0.784	0.651
	F ₁₇₉	F ₇₃₂	0.784	0.675
	F ₃₁₈	F ₄₁₉	0.728	0.793
	F ₃₁₈	F ₇₁₅	0.728	0.651
	F ₄₁₉	F ₇₁₅	0.793	0.651
	F ₄₁₉	F ₇₃₂	0.793	0.675
Flavor	F ₅₇₁	F ₇₁₅	0.744	0.651
	F ₆₆₁	F ₇₁₅	0.742	0.651
	F ₇₁₅	F ₈₇₃	0.651	0.747
	F ₇₃₂	F ₈₇₃	0.675	0.747
	F ₁₇₉	F ₇₁₅	0.786	0.654
	F ₁₇₉	F ₇₃₂	0.786	0.678
	F ₃₁₈	F ₄₁₉	0.730	0.795
	F ₃₁₈	F ₇₁₅	0.730	0.654
	F ₄₁₉	F ₇₁₅	0.795	0.654
	F ₄₁₉	F ₇₃₂	0.795	0.678
	F ₅₇₁	F ₇₁₅	0.746	0.654
	F ₅₇₁	F ₇₃₂	0.746	0.678
	F ₆₆₁	F ₇₁₅	0.744	0.654
	F ₇₁₅	F ₈₇₃	0.654	0.749
	F ₇₃₂	F ₈₇₃	0.678	0.749

The pairwise comparisons summarized in Table 2 revealed a consistent pattern of sensory acceptance across the five attributes evaluated. Among the eight formulations, F_{419} and F_{179} were consistently rated highest with significantly higher mean acceptance scores than F_{715} and F_{732} across all sensory attributes. Specifically, formulation F_{419} achieved superior evaluations in color, flavor, and sweetness, with posterior mean scores exceeding 0.79, while F_{179} distinguished itself in acidity, aroma, and color, consistently outperforming other samples in these dimensions. Conversely, formulations F_{715} and F_{732} were recurrently associated with the lowest acceptance scores, appearing as the inferior alternatives in the majority of significant pairwise comparisons. This performance gap was particularly pronounced compared with formulations F_{419} , F_{179} , and F_{873} , which demonstrated markedly higher sensory acceptance across the evaluated attributes, as shown in Figure 1.

Overall, the results underscore a clear distinction in sensory acceptance among the eight grape juice formulations evaluated. Specific formulations consistently outperformed others, evidencing perceptible differences in consumer preferences across the assessed sensory attributes.

Additionally, the posterior standard deviations of the attribute-specific random effects, reported in Table 3, were derived from the diagonal elements of the posterior covariance matrix Σ , obtained by inverting the precision matrix Ω . These estimates quantify the variability in sensory perception attributed to each attribute, reflecting heterogeneity across panelists. Among the attributes, flavor exhibited the highest variability ($\hat{\sigma} = 1.02$), whereas aroma presented the lowest variability ($\hat{\sigma} = 0.88$). These findings reinforce the need to incorporate random effects for attributes within the model structure to capture such intrinsic differences in perception.

Table 3. Posterior means and 95% HDIs for the standard deviations of the random effects associated with each sensory attribute, according to the sensory experiment performed at ESALQ/USP in 2025

Attribute	Mean ($\hat{\sigma}_a$)	95% HDI Lower	95% HDI Upper
Color	0.89	0.28	1.91
Aroma	0.88	0.29	1.90
Flavor	1.02	0.28	2.23
Sweetness	0.90	0.27	1.96
Acidity	0.89	0.26	1.90

The posterior covariance matrix Σ , which captures the variability and potential dependencies among sensory attributes, was obtained as part of the Bayesian estimation process. Specifically, samples were drawn from the Wishart prior assigned to the precision matrix $\Omega = \Sigma^{-1}$ during model specification. After verifying convergence, these samples were inverted to yield posterior draws of Σ , enabling the recovery of both variances and covariances associated with attribute-level random effects.

To facilitate the interpretation of inter-attribute relationships, the covariance matrix Σ associated with the attribute-level random effects was standardized, yielding the posterior correlation matrix shown in Table 4. The results indicate moderate positive correlations among several attribute pairs. Notably, flavor exhibited the strongest associations, particularly with color ($\hat{\rho} = 0.229$) and sweetness ($\hat{\rho} = 0.222$), suggesting that these attributes share overlapping perceptual components in the evaluation of grape juice.

Beyond statistical significance, these findings offer direct applications for the food industry, providing clear guidance for product development. Formulations F_{419} (natural) and F_{179} (processed) emerged as consistently superior across multiple sensory attributes, indicating that their flavor profiles should be prioritized. A strong association between flavor and color supports the notion that visual characteristics can influence taste perception, as previously demonstrated by studies showing that color attributes modulate the perceived intensity and preference for flavor, aroma, and acidity in

beverages (Spence, 2019; Kardas *et al.*, 2024; Fernández-Vázquez *et al.*, 2014). Such evidence should be incorporated into future product development, with an emphasis on visual qualities—particularly color—as a driver of sensory perception, given its influence primarily on flavor and aroma. This approach may enhance consumer expectations, contribute to a balanced sensory profile, and ultimately increase consumer engagement and boost sales, as also discussed by Kardas *et al.* (2024) in their study on beetroot and tomato juices.

These posterior correlations emphasize the importance of modeling sensory attributes within a correlated multivariate random effects framework, as it captures latent perceptual linkages that influence acceptance ratings. By explicitly accounting for these interdependencies, the model provides a more comprehensive representation of consumer perception, facilitating more precise formulation strategies. The estimated correlation matrix quantifies the strength and direction of associations among sensory attributes, offering a rigorous basis for understanding the multivariate structure of consumer evaluations. These findings highlight the added value of Bayesian hierarchical modeling in enabling joint inference on both individual attributes and their interrelationships.

Table 4. Posterior correlation matrix among the random effects of the sensory attributes, based on the study conducted at ESALQ/USP in 2025

	Color	Aroma	Flavor	Sweetness	Acidity
Color	1.000	0.149	0.229	0.156	0.138
Aroma	0.149	1.000	0.213	0.167	0.108
Flavor	0.229	0.213	1.000	0.222	0.207
Sweetness	0.156	0.167	0.222	1.000	0.144
Acidity	0.138	0.108	0.207	0.144	1.000

The model's predictive accuracy was subsequently evaluated using the LOO-CV criterion, derived from pointwise log-likelihood calculations. The estimated ELPD was $elpd_{100} = 1814.7$ with an associated standard error of 92.7, while the effective number of parameters was estimated at $p_{100} = 117.4$ (SE = 6.5). The robustness of these estimates was further supported by Pareto- k diagnostics, which indicated that all observations yielded k values below the recommended threshold of 0.7, ensuring the reliability of the approximation.

Although a direct comparison of inferential methodologies was not the primary objective of this study, the Bayesian framework enabled complementary analyses such as the estimation of correlations among sensory attributes. Nevertheless, the formulations identified as the most well-rated, notably F_{419} and F_{179} , as well as those least preferred, such as F_{732} and F_{715} , were the same as those reported by Alves *et al.* (2025) using a classical inference approach. This consistency reinforces the reliability of the selected formulations and highlights the robustness of the findings across distinct statistical paradigms.

Additionally, to assess the effects of correlations among sensory attributes and random effects, a predictive comparison was performed using PSIS-LOO and the Watanabe-Akaike Information Criterion (WAIC) among three model specifications: (i) the full model, with correlations among attributes and hierarchical random effects (Equation 5); (ii) a model similar to the previous one but assuming independence among attributes, given by $\text{logit}(\mu_{ijl}) = \eta_{ijl} = b_0 + \beta_j + \beta_l + u_i$; and (iii) a model without hierarchical random effects, defined as $\text{logit}(\mu_{ijl}) = \eta_{ijl} = b_0 + \beta_j + \beta_l$. The results indicated nearly identical performance between models (i) and (ii), with $elpd_{loo}$ values of 1814.23 and 1813.96, and their difference, representing the gain or loss in predictive ability, denoted by $\Delta elpd \approx -0.3$. Model (iii), in turn, showed poorer predictive performance, with $elpd_{loo} = 1729.75$ and $\Delta elpd \approx -84.5$ relative to model (i). The WAIC values confirmed these findings, with -3681.1 , -3679.2 , and -3482.5 for models (i), (ii), and (iii), respectively. These results demonstrate that hierar-

chical random effects are essential to capture individual variability among panelists and to improve predictive quality. In contrast, the correlation structure among attributes, although not substantially altering elpd_{100} , contributes to representing sensory interdependencies and providing a more coherent joint interpretation of the data. It is important to note, however, that these conclusions refer to the empirical context analyzed and may vary according to the data and experimental design adopted.

5. Discussion

The interpretation of the results from the Bayesian beta model reveals patterns consistent with evidence previously documented in studies using different types of juices. In particular, the positive contribution of sweetness and aroma to overall acceptance, together with the more discrete role of acidity, corroborates findings indicating that hedonic responses to grape juice are driven by congruent flavor and aroma cues and by moderate levels of sweetness (Gous *et al.*, 2019). Moreover, as highlighted by Matsuura *et al.* (2002) and Cruz *et al.* (2018), color exerts a strong influence on product attractiveness, since more saturated tones tend to intensify flavor perception and increase acceptance in beverages. Taken together, these findings indicate that consumer acceptance results from the integration of multiple correlated sensory channels rather than the isolated evaluation of a single attribute.

In addition to its coherence with the sensory literature, the results also emphasize the practical advantages of the adopted hierarchical Bayesian model. This approach provides greater flexibility for fitting complex models, even in scenarios with a large number of parameters or sample restrictions imposed by the experimental design. Furthermore, Bayesian inference allows uncertainty to be characterized more directly and intuitively, expressing it in terms of probabilistic distributions over the parameters themselves, thereby enhancing the interpretability of the results while facilitating the incorporation of potential multilevel structures (Flores *et al.*, 2022).

From a computational standpoint, unlike confidence intervals derived from the classical approach, which depend on asymptotic approximations, Bayesian intervals maintained satisfactory performance and directly express the probability that the parameter lies within a given range (Dunson, 2001). Thus, the method provides a robust alternative to maximum likelihood estimates, reconciling statistical rigor with practical interpretability.

From a practical standpoint, the proposed hierarchical mixed beta model can assist product development decisions by allowing multiple sensory attributes to be evaluated simultaneously under realistic experimental constraints. This is particularly relevant in industrial sensory studies, where balanced incomplete block designs are commonly used to limit panelist workload and experimental costs. By borrowing information across attributes and panelists, the model yields more stable estimates and facilitates clearer formulation comparisons than analyses conducted separately for each attribute.

5.1 Limitations

The proposed framework, while effective, has some constraints. First, the logit-linear predictor assumes additive effects on the logit scale, which may fail to capture nonlinear or interaction effects that can arise in extreme hedonic responses (e.g., when panelists consistently rate certain formulations near the scale minimum or maximum). Second, performance with small panels (< 30) remains untested and may require informative priors. Third, very high attribute correlations (> 0.8) could challenge covariance matrix identifiability. These limitations, however, are offset by the model's advantages in handling bounded data and panelist heterogeneity, and they suggest clear paths for future extensions.

6. Conclusion

This study has demonstrated the successful application of a hierarchical Bayesian beta regression model to sensory evaluation data, yielding consistent conclusions regarding formulation preferences providing a robust alternative to classical maximum likelihood approaches, that may present limitations. The proposed methodology offers several substantive advantages for sensory science applications, particularly through its capacity to model complex dependence structures inherent in multidimensional hedonic assessments.

The Bayesian framework proved particularly valuable in three key aspects of the analysis. First, the hierarchical structure naturally accommodated random effects at both the panelist and attribute levels, enabling proper quantification of individual variability in sensory perception. Second, the multivariate specification of attribute effects revealed meaningful correlations among sensory dimensions, with particularly strong associations observed between flavor, color, and sweetness ($\hat{\rho} \approx 0.22$). These findings corroborate existing psychophysical understanding of integrated sensory perception while providing quantitative estimates of these relationships. Third, it was possible to include random effects in the dispersion parameter, capturing individual-level variation among panelists and improving the model's capacity to represent heterogeneity in sensory perception.

The inclusion of a multivariate structure for the attribute-level random effects proved essential for capturing perceptual interactions among sensory dimensions. Posterior correlations revealed moderate positive relationships, particularly between flavor, color, and sweetness, which align with overlapping hedonic constructs typically observed in grape juice evaluation. These findings emphasize the relevance of modeling correlated attribute effects in sensory studies, thereby enabling more accurate interpretations and offering valuable guidance for formulation optimization and product development.

Several promising directions for methodological extensions of this work emerge. Future research could investigate alternative prior specifications and their impact on inference robustness; the use of zero- and one-inflated beta distributions to address boundary-inflated sensory data; the application of multivariate Dirichlet regression approaches for compositional attribute analysis; the adoption of nonlinear link functions to capture potential threshold effects in hedonic scaling; and extensions to longitudinal or dynamic sensory settings, allowing repeated evaluations over time to be explicitly modeled within the hierarchical framework.

The current implementation already provides sensory scientists with a powerful tool for analyzing complex hedonic data, offering advantages over conventional methods through its proper handling of random effects, correlated attributes, and bounded response distributions. As the field moves toward more sophisticated analyses of consumer perception data, such Bayesian approaches will likely play an increasingly important role in bridging the gap between statistical modeling and sensory science applications.

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Conflicts of Interest

The authors declare that there is no conflict of interest in this work.

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7. Appendix

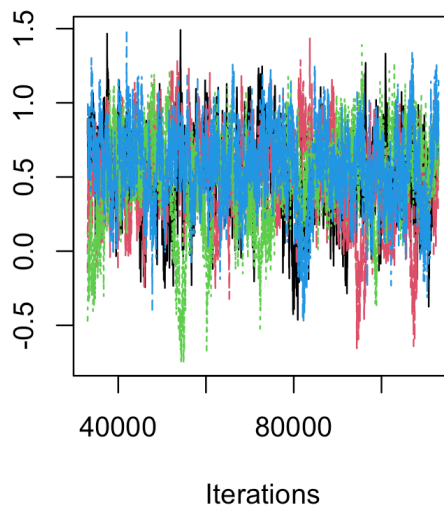


Figure 2. Trace plot of MCMC posterior draws for parameter b_0 , estimated from the sensory evaluation data collected at ESALQ/USP (2025).

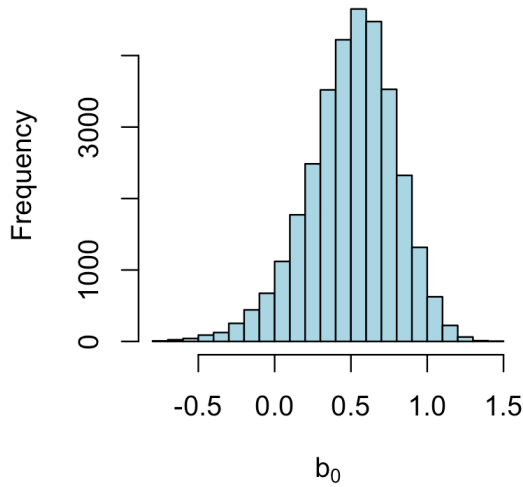


Figure 3. Estimated posterior distribution of parameter b_0 , derived from Bayesian inference based on sensory data obtained at ESALQ/USP (2025).

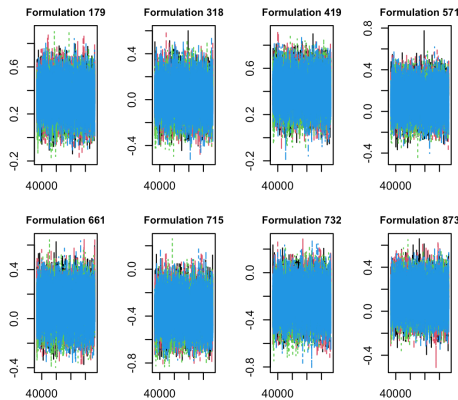


Figure 4. Trace plots of MCMC posterior draws for the parameters β_{form} , representing the formulation effects estimated from the sensory evaluation data collected at ESALQ/USP (2025).

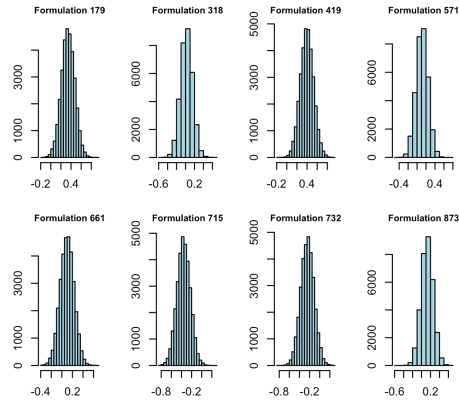


Figure 5. Estimated posterior distribution of parameter β_{form} , derived from Bayesian inference based on sensory data obtained at ESALQ/USP (2025).

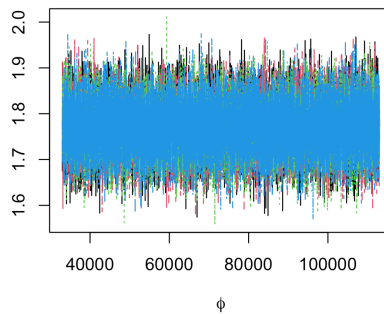


Figure 6. Trace plots of MCMC posterior draws for the parameter ϕ derived from the sensory evaluation data collected at ESALQ/USP (2025).

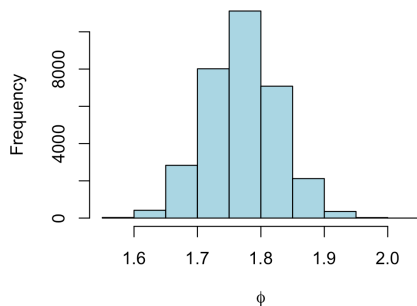


Figure 7. Estimated posterior distribution of parameter ϕ , derived from Bayesian inference based on sensory data obtained at ESALQ/USP (2025).

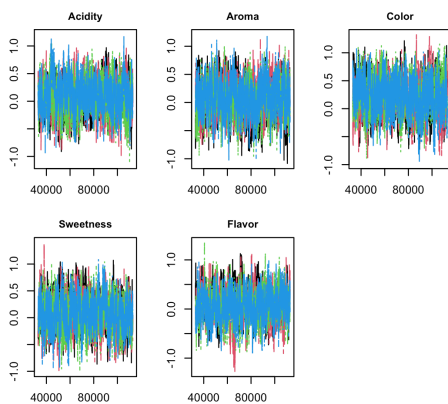


Figure 8. Posterior distributions of the fixed effects associated with sensory attributes, based on the sensory evaluation data collected at ESALQ/USP (2025).

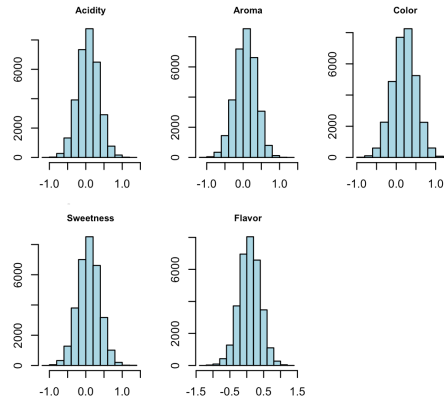


Figure 9. Estimated posterior distribution of the fixed effects related to sensory attributes, based on the sensory evaluation data collected at ESALQ/USP (2025).

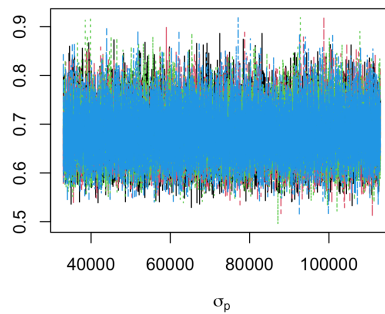


Figure 10. Posterior distribution of the standard deviation σ_p of the panelist random effects, based on the sensory evaluation data collected at ESALQ/USP (2025).

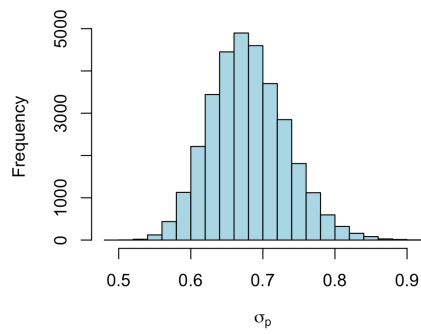


Figure 11. Trace plot of MCMC posterior draws for the standard deviation σ_p of the panelist random effects, based on the sensory evaluation data collected at ESALQ/USP (2025).