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ARTICLE

A probabilistic model for waiting time in achieving family planning goals[1]

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Abstract
The family decision process is a complex phenomenon. At some point, a couple may decide to interfere with the childbirth process in some way. The result of this decision is a sudden event that limits family size and gender composition. An important consideration in a study dealing with the number of children families wish to have is whether these desires include preference as to the child's sex. The purpose of this paper is to propose five hypothetical rules that reflect the current preferences of parents regarding the size and gender composition of their children. Various combinations of marital durations, levels of fecundity, and rest periods (gestational period plus amenorrhea period) have been used to calculate the expected waiting time for each case to reach the desired family size. Additionally, for each case, an estimate of the truncation bias has been obtained. The lower limit of the range in each case is constructed by that value of the expected waiting time which corresponds to the pair of parameters $\lambda = 0.60$ and $h = 0.92$ years. The upper limit is the value of the expected waiting time derived by assuming $\lambda = 0.40$ and $h = 1.25$ years for an infinite duration of the marriage. As a result of this information, couples may be able to plan their families in such a way that all their financial and social obligations have been met by the time they plan to retire, which means that it will help make decisions about family planning at the micro level.

Keywords: Probability Model; Truncation Bias; Stopping Rules; Waiting time

1. Introduction

The family decision-making process is not as straightforward as other physical or biological processes. Several factors, such as the number of surviving children, the sex composition, cultural norms, socioeconomic status, the utility of childbearing, the subjective evaluation of contraception, etc., act simultaneously in decision-making. In developing countries, where infant and child mortality rates are quite high, couples would like to ensure the survival of their minimum desired number of sons and daughters before deciding to end their childbearing process. Krishnamoorthy (1974) derived the distribution of the surviving children on the assumptions that (i) infant and child mortality occurs in the first two years of life and later no death occurs among children until the couples complete their fertility, and (ii) the couple decides to have another child when the previous child turns two years old. A couple may at some point decide to interfere with the birth process. The consequence of this decision is a chance phenomenon leading to a finite size and sex composition of the family. Knowledge of the expected time required to accomplish the intended family size and sex composition would be useful for micro-level decision-making. Couples could use this information to plan their families in such a way that their social and financial liabilities are satisfied before they retire from active life. The utility of contraception may also be better realized by individual couples if they are aware of the chance mechanism of the family building process and the expected time to achieve the desired size and sex composition of the family (Tiwari and Saroj, 2025; Bhatnagar *et.al., 2025;* Maurya *et.al., 2025*). This information may prove beneficial in formulating national population policy. Bearing in mind the utility of the information we need and the complexity that it poses, Pathak and Saxena (1979) devised a mathematical model that may assist answer the concerned queries at the micro as well as macro level without creating much mathematical complication, and for illustrative purposes, four hypothetical cases, called "stopping rules," have been investigated. In this paper, an attempt has been made to propose five new hypothetical rules that reflect the current preferences of parents regarding the size and sex composition of children, and for each case, the expected time for attaining the desired family size has been computed for various combinations of marital durations, levels of fecundability, and rest periods (gestation period plus amenorrhea period). Also, an estimate of the truncation bias (introduced due to short marital duration) has been obtained for each case.

2. The Problem and Its Analogy

Several KAP (knowledge, attitudes, and practices) studies have revealed that in less developed nations with a low literacy rate, the "ideal family size" reported by a couple is a function of the size they actually had at the time of the survey. The true option regarding the "desired family size" is observed to be time-dependent and subject to the change in the utility of childbearing to the couple as a result of the sex composition attained or the change in socioeconomic status. In a survey, it may be challenging to determine the ideal and desirable family sizes based on the responses of couples. A preference related to some generic ideal or norm may not be equivalent to a signal for action in practice. Since desires change over time, a preference held prior to the birth of children may shift after the birth of children. ‘how many kids do you want?’ A frequently included question in surveys relates to the number of children people actually have. Not infrequently, expressed desires correlate poorly with the ultimate size of families. Even among those with high fertility, the preference is often to have two, three, or four children. Generally speaking, it is known that responses to questionnaires or interviewers may not have a high degree of reliability. This is true even for relatively objective data without the emotional connotations inherent in children's wishes. Regarding child planning, Westoff *et al.* (1961) reported notable inconsistencies in interviews approximately three years apart.

This can be determined, however, if the couples' intended sex composition of their offspring is known. Keeping this in mind, Sheps (1963) used the well-known results of the coin-flipping experiment to demonstrate the effect of sex preference on the completed family size by assuming that a woman's reproductive life is infinite. Mitra (1970) has derived such expressions when the strategy is to have *m* sons and *f* daughters in no more than k trials. As such, the couples may stop at birth of the $(k-l)^{th}$ child $(l>0, m+f \le k-l)$ when either the couple achieve the desired sex composition of children or the sex of the $(k-l)^{th}$ child makes it impossible for the couple to achieve the goal. Krishnamoorthy (1974) extended earlier findings by taking into account the fact that couples require a minimum of *g* children and then want to satisfy their desire of *m* sons and *f* daughters in *k* births. The question now is whether a woman can conceive as often as she desires. Contextually, we know that (1) the duration of her fertile life is limited; (2) her conceptions are random; (3) a portion of her reproductive life is wasted due to postpartum ammenorrhea periods; and (4) a couple may stop having children voluntarily; this may be due to the number and sex composition of the born children and a sense of being unable to achieve the desired composition. Thus, the conditions for a coin-tossing experiment as described by Sheps (1963) and Mitra (1970) are not fully met by the process of reproduction. With this idea, Pathak (1973) proposed a model to study the variation in family size achieved over a given period of time under different preferences of parents with respect to the sex of children. Pathak and Saxena (1979) have developed a model for the waiting time for the realization of such an event using a similar approach. Obviously, its generalized version could be obtained on the lines of Das Gupta (1973) and Mode (1975) to be modified because the martial duration is finite and trials are limited. Several attempts have been made to analyze the consequences of preference for boys in populations using birth control in which couples have different probabilities of producing boys (Goodman, 1961; McDonald, 1973; Mode, 2012). In the present study, with the help of a probability model given by Pathak and Saxena (1979), some estimated results have been obtained for studying the expected waiting time under various stopping rules adopted by the parents based on the size and sex composition of their family.

The demographic problem considered in this paper can be compared to the one of life testing in the case of certain engineering products, where the experimenter is interested in observing n articles for a period of time T and stopping the experiment when either $r \le n$ failures occur or the time T elapses. One of the variables of interest is the waiting time for the r^{th} failure.

The problem necessarily reduces to determining the waiting time distribution, which is both truncated in time and censored for the number of failures. In the present case, however, the problem of obtaining the waiting time distribution is not so straightforward: after a conception, there is no chance of another conception for some time due to the woman's rest period; as soon as she is released from confinement and the associated amenorrhea period, her waiting time begins. These states recur. In the case of engineering goods, however, it is not the same item that fails repeatedly; rather, the experimenter is interested in r failures out of n items before time *T*. Here our interest lies in finding the waiting time for realizing any one condition satisfying the desire of the couple with respect to their family size before time *T*. The distribution described below can also be used in cases analogous to the life test.

3. The Model

The underlying assumptions of the model given by Pathak and Saxena (1979) are as follows:

1. A woman is not pregnant and is fecund at the time of consummation of her marriage and continues to be in marital union until time *T*.
2. The probability that $(i+1)^{th}$ conception occurs during the time interval $(t, t+dt)$ is $\lambda dt + O(dt)$ if the i^{th} conception occurs prior to $t-h$, where $t \geq ih; \lambda > 0$, and zero otherwise; *h* is the rest period associated with a live birth conception.
3. Let $(1-\alpha_i)$ be the probability that a woman becomes secondary sterile after attaining the i^{th} parity.
4. The probability that a born child will be male is *g* and female (1-*g*). The probability of twin or multiple births is zero.
5. A couple decides to stop whenever any one of the following conditions are fulfilled: (i) either *a* sons and *b* daughters are born, or (ii) $m\left(m > a+b\right)$ children are born.

Undoubtedly, some of these assumptions are robust, but they may be viewed as a first approximation of the actual process.

Let *x* represent the time required by a couple to achieve the desired family size and sex composition. Suppose this is achieved at parity *i*, then the joint distributions of *x* and *i* is given by:

$$f(x,i) = \prod_{j=1}^{i-1} \alpha_j \frac{\lambda^i}{(i-1)} \left[x - G - (i-1)h\right]^{(i-1)} \exp\left[-\lambda\left\{x - G - (i-1)h\right\}\right]. \left[\binom{i-1}{a-1} g^a (1-g)^{i-a} + \binom{i-1}{b-1} g^{i-b} (1-g)^b\right] \quad (1)$$

where, $x \geq (i-1)h + G$

and $i = (a+b), (a+b+1), ..., (m-1)$

G is gestation period.

Now, since the couple stops whenever m children are born according to assumption (2), the joint distribution of x and m is given by:

$$f(x,i) = \prod_{j=1}^{m-1} \alpha_j \frac{\lambda^m}{(m-1)} \left[x - G - (m-1)h\right]^{(m-1)} \exp\left[-\lambda\left\{x - G - (m-1)h\right\}\right]. \left[\sum_{k=0}^{a-1} \binom{m-1}{k} g^k (1-g)^{m-k-1} + \sum_{k=0}^{b-1} \binom{m-1}{k} g^{m-k-1} (1-g)^k\right] \quad (2)$$

where, $x \geq (i-1)h + G$

The mean waiting time $\bar{x}$ is:

$$\bar{x} = \frac{\sum_{i=a+b}^{m-1} \int_{(i-1)h+G}^{T} xf(x,i)\,dx + \int_{(m-1)h+G}^{T} xf(x,m)\,dx}{\sum_{i=a+b}^{m-1} \int_{(i-1)h+G}^{T} f(x,i)\,dx + \int_{(m-1)h+G}^{T} f(x,m)\,dx} \quad (3)$$

It may, however, be noted that here X is not a proper random variable because all the couples cannot attain their desired family size within the period $(0,T)$. However, it may be noted that here X is not a proper random variable as not all couples can achieve their desired family size within the period $(0,T)$ .For numerical values of T, h, G, and m, both the numerator and the denominator can actually be easily found after integrating definite integrals.

The variance of the waiting time X can be found by finding the second moment (μ_2') about the origin, as below:

$$\mu_2' = \frac{\sum_{i=a+b}^{m-1} \int_{(i-1)h+G}^{T} x^2 f(x,i)\,dx + \int_{(m-1)h+G}^{T} x^2 f(x,m)\,dx}{\sum_{i=a+b}^{m-1} \int_{(i-1)h+G}^{T} f(x,i)\,dx + \int_{(m-1)h+G}^{T} f(x,m)\,dx} \quad (4)$$

When $T = \infty$, the expression (3) reduces to:

$$\bar{x} = \frac{\sum_{i=(a+b)}^{m-1} \left\{\frac{i}{\lambda} + G + (i-1)h\right\} P_i + \left\{\frac{m}{\lambda} + G + (m-1)h\right\} P_m}{\sum_{i=(a+b)}^{m-1} P_i + P_m} \quad (5)$$

The values of P_i and P_m can be computed from the expressions given by the Sheps (1963).

P_i = Probability that the desire of the couple is fulfilled at the birth of the i^{th} child

$$= \left[\binom{i-1}{a-1} g^{a} (1-g)^{i-a} + \binom{i-1}{b-1} g^{i-b} (1-g)^{b} \right] \quad (6)$$

P_m = Probability that the desire of a couple with respect to the size and sex composition of family is not fulfilled till the birth of $(m-1)^{th}$ child and consequently the woman stops reproduction when m^{th} child is born

$$P_m = \left[\sum_{k=0}^{a-1} \binom{m-1}{k} g^{k} (1-g)^{m-k-1} + \sum_{k=0}^{b-1} \binom{m-1}{k} g^{m-k-1} (1-g)^{k} \right] \quad (7)$$

Also, for $T = \infty$, the expression (4) reduces to:

$$\mu_2' = \frac{\sum_{i=(a+b)}^{m-1} \left[\frac{i(i+1)}{\lambda^2} + \frac{2i\{G+(i-1)h\}}{\lambda} + \{G+(i-1)h\}^2 \right] P_i}{\sum_{i=(a+b)}^{m-1} P_i + P_m} + \frac{\left[\frac{m(m+1)}{\lambda^2} + \frac{2m\{G+(m-1)h\}}{\lambda} + \{G+(m-1)h\}^2 \right] P_m}{\sum_{i=(a+b)}^{m-1} P_i + P_m} \quad (8)$$

Expressions (5) and (8) have been derived by first finding the $E(x/i)$ and $E(x^2/i)$ and then by taking the expectations of these conditional measures.

3.1 Illustration

Five hypothetical cases, giving the rules when a couple would stop, have been considered, and in each case the expected time for achieving the desired family have been computed for various combinations of marital duration (*T*), level of fecund ability (λ), and the rest period (*h*). These five rules are, however, chosen arbitrarily and serve only as examples. Various other stopping rules can be created and expected waiting times can be calculated. But we limit the scope of this analysis to the five cases listed below:

Rule1: A couple stops when at least one son or two children are born (*a*=1, and *m*=2).

Rule2: A couple stops when two children are born (*m*=2).

Rule3: A couple stops when at least one son and one daughter or three children are born (*a*=1, *b*=1, and *m*=3).

Rule4: A couple stops when three children are born (m=3).

Rule5: A couple stops when two son and one daughter or four children are born (a=2, b=1, and m=4).

Using the expressions (1), (2) and (3) given in the previous section, the expected values of the waiting time for fulfilling the desire of couples in the above cases have been computed corresponding to T=10 and 15 years, h = 0.92 or 1.25 years and λ = 0.40 or 0.60. The values of λ have been chosen arbitrarily but they are consistent with the empirical estimates derived by Singh (1968), Singh and Pathak (1968) and Saxena (1969) for Indian women. The values of rest period h include nine months of gestation plus two months or six months of postpartum amenorrhoea. For simplicity we have assumed g= ½. Though in reality $g \neq ½$ and also may vary from parity to parity, for the computation of the expected waiting time, g = ½ is a reasonable approximation since a slight deviation from this value will not have significant effect on the results. Further, we have assumed $\alpha_i = \alpha$ and the empirical value of α is taken as 0.96 as obtained by Pathak (1975). The expected waiting time under different sex preferences and stopping rules are presented in Table 1.

The standard deviations of the waiting time have been computed under different rules for $T = \infty$, from the expressions (5) and (8). The results are presented in Table 2. It may be mentioned, however, that the standard deviation of the waiting time under a finite marital duration would be less than its corresponding value computed for $T = \infty$.

3.2 Truncation Bias

Sheps (1963) examined the effect of sex preference on the size of the completed family by assuming the reproductive life of a woman is infinite. However, a woman's fertile period is limited, and therefore, it would be more appropriate to consider it finite.But, again, for a short marital duration, only those couples will be satisfied in terms of desired family size and gender composition, whose female spouses are likely to conceive early, and thus a bias in the estimation of the expected waiting time is introduced. The shorter the marital duration, the greater the extent of bias (Sheps *et al.*, 1970).To make the results in Table1 more meaningful, it is necessary to examine the effect of truncating the duration of marriage on the expected waiting time. The estimate of the extent of the bias in each case is obtained by subtracting the value of the expected waiting time computed for the finite marital duration (T) from the value computed for the infinite marital duration for the given set values of the parameters λ and h.

4. Results and Discussion

Table 1. Expected Waiting Time under Different Sex Preferences and Stopping Rules

Marital Duration T (In Years)	Fecundability λ	Rest Period h (In Years)	Expected Waiting Time $\bar{x}$ (in years)				
			Rule 1	Rule 2	Rule 3	Rule 4	Rule 5
10	0.40	0.92	4.59	5.50	5.60	7.40	7.80
		1.25	4.73	5.74	5.85	7.78	8.06
	0.60	0.92	3.91	4.71	4.88	6.47	7.27
		1.25	4.10	5.00	5.19	7.01	7.68
15	0.40	0.92	5.26	6.32	6.47	8.59	9.91
		1.25	5.45	6.61	6.78	9.17	10.46
	0.60	0.92	4.11	4.97	5.14	6.75	8.18
		1.25	4.33	5.29	5.50	7.40	8.97
∞	0.40	0.92	5.53	6.67	6.82	8.84	10.83
		1.25	5.75	7.00	7.17	9.50	11.68
	0.60	0.92	4.14	5.00	5.17	6.76	8.26
		1.25	4.36	5.33	5.54	7.42	9.11

It may be seen that the expected waiting time for achieving the desired family size under Rule 5 is higher as compared to the remaining four hypothetical cases shown in Table 1.In this case, a greater preference is shown for boys, a couple may stop reproduction even after three children, provided the desired sex composition is achieved in the family.Under Rule 4, no indication of sex preference is shown as well as a couple's desire is fulfilled when three children are born. So, the expected waiting time for this rule is less as compared to the expected waiting time for Rule 5.In accordance with Rule 3, however, where a couple gives equal preference for a child of each sex, a couple may stop reproduction even after two children provided the desired sex composition in the family has been attained, and hence, the expected waiting time, in this case, is less than to obtain three children.Under Rule 2, no sex preference is indicated and also the desire of a couple is fulfilled when two children are born. Hence the expected waiting time is shorter than the expected waiting time obtained under Rules 3, 4, and 5.It is least for Rule 1, in which the desire for a girl is not expressed and a couple may stop reproduction after the birth of one son. Table 1 also reveals that couples whose female partners are more fecund and have shorter periods of postpartum amenorrhoea achieve satisfaction sooner in relation to the size and sex composition of the family than those with lower fecundability and higher duration of postpartum amenorrhoea.

Table 2. Standard Deviations of Waiting Time of Satisfied Couples with Infinite Marital Duration under Different Sex Preferences and Stopping Rules

Fecundability λ	**Rest Period** h **(In Years)**	**Standard Deviations of Waiting Time (In Years) of Satisfied Couples Under**				
		Rule 1	**Rule 2**	**Rule 3**	**Rule 4**	**Rule 5**
0.40	0.92	3.31	3.24	4.21	4.21	5.07
	1.25	3.38	3.24	4.32	4.24	5.09
0.60	0.92	2.37	2.16	3.14	3.03	3.15
	1.25	2.45	2.23	3.25	3.02	3.14

Table 2 gives standard deviations of waiting time of satisfied couples with infinite marital duration under different sex preferences and stopping rules and Table3 gives the estimates of truncation bias in the expected time for attaining the desired family size under different sex preferences and stopping rules.

Table 3. Estimates of Truncation Bias in the Expected Time for Attaining the Desired Family Size under Different Sex Preferences and Stopping Rules

Marital Duration T (In Years)	Fecundability λ	Rest Period h (In Years)	Truncation Bias (in years) in the Expected Waiting Time under				
			Rule 1	Rule 2	Rule 3	Rule 4	Rule 5
10	0.40	0.92	0.94	1.17	1.22	1.44	3.03
		1.25	1.02	1.26	1.32	1.72	3.62
	0.60	0.92	0.23	0.29	0.29	0.29	0.99
		1.25	0.26	0.33	0.35	0.41	1.43
15	0.40	0.92	0.27	0.35	0.35	0.25	0.92
		1.25	0.30	0.39	0.39	0.33	1.22
	0.60	0.92	0.03	0.03	0.03	0.01	0.08
		1.25	0.03	0.04	0.04	0.02	0.14

It is evident from Table 3 that the extent of bias due to truncation is smaller for couples whose female spouses are more fecund and whose average duration of postpartum amenorrhea is shorter. Also, the higher the desired size of the family, the greater is the extent of the bias. For example. the bias is least under Rule 1 when$\lambda = 0.60$and$h$ = 0.92, for a given marital duration T. Due to the fact that the only requirement in this instance is to have a son, the majority of couples have their desires fulfilled much earlier, with only a small percentage remaining unsatisfied. Evidently, couples whose female partners are quick conceivers have been included, and as a result, the extent of bias due to truncation is greater in cases with stricter requirements.

5. Conclusions

The possible range of the expected waiting time to satisfy the couple's desire with respect to family size and gender composition can be easily read in Table 1. Thus, a more accurate estimate of future needs for family planning services can be obtained if all marriages are registered and at the time of registration, the wishes of the couple regarding the size and sex composition of the family they desire are recorded. Couples who may require family planning services at a particular time or over a period of time could be identified through the use of this a priori information along with the expected waiting time of having the desire fulfilled.This selective approach would not only reduce the operational costs of family planning administration, but it would also maximize the returns in terms of sterilization, number of IUD insertions, etc. in relatively less time and with greater efficiency.

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Conflicts of Interest

No potential conflict of interest was reported by the author(s).

Author Contributions

Conceptualization: SINGH P. B.; SINGH T. **Data curation:** SINGH P. B.; SINGH T.; SINGH K. A. **Formal analysis:** SINGH P. B.; SINGH T.; SINGH K. A. **Methodology:** SINGH P. B.; SINGH T.; SINGH K. A. **Software: - Resources:** SINGH T.; SINGH K. A. **Supervision:** SINGH P. B.; SINGH T. **Validation:** SINGH P. B.; SINGH T.; SINGH K. A. **Visualization:-Writing - original draft:** SINGH T. **Writing - review and editing:** SINGH P. B.; SINGH T.; SINGH K. A.

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