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ARTICLE

On optimal estimation techniques for supply chain management with stratified sampling[1]

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Abstract
In this study, improving the estimation of the population mean is critical in supply chain management for optimizing resource allocation, determining the optimum sample size, and enhancing operational efficiency. This study proposes an improved method for estimating the population mean within a stratified random sampling framework by incorporating an auxiliary variable to increase the precision of delivery time estimates and minimize costs incurred. Population data are generated under a bivariate normal distribution across four stratified regions, using shipment volume as an auxiliary variable correlated with delivery time. A first-order error approximation is employed to derive an efficient estimator, which reduces bias and improves accuracy. A simulation study with a proportionally allocated stratified sample is conducted to evaluate the performance of the proposed estimator. The results demonstrate reduced variation and increased estimation efficiency, and cost-effectiveness compared to traditional mean estimators. This approach provides robust statistical insights for logistics companies, enabling data-driven decision-making and enhanced supply chain performance.
Keywords: Supply Chain Management; Stratified Sampling; Delivery time; Bivariate Normal Distribution; sample Size; Cost Incurred

1. Introduction

Sampling is a statistical technique used to select a subset of individuals or observations from a larger population to make inferences about the whole group. It is widely applied in survey research, quality control, medical studies, and supply chain management to save time and costs while maintaining accuracy. Sampling methods are generally categorized into probability and non-probability sampling. Probability sampling ensures that each unit has a known, non-zero chance of selection, thereby reducing bias and improving representativeness, with common methods including Simple Random Sampling (SRS), Stratified Random Sampling, Systematic Sampling, and Cluster Sampling. In contrast, non-probability sampling relies on convenience or judgment rather than randomness and includes methods such as convenience sampling, judgmental (purposive) sampling, quota sampling, and snowball sampling. The choice of method depends on research objectives, available resources, and the desired level of precision.

Hansen *et al.* (1946) discusses challenges in conducting sample surveys for business statistics, focusing on sample design, data collection, estimation techniques, and error sources. They emphasized the importance of proper stratification, weighting, and adjustment methods for accuracy and laid the foundation for modern business survey methodologies. Cochran (1977) study the survey sampling theory and methodology, covering various techniques like simple random, stratified, systematic, cluster, and multistage sampling. It provides theoretical justifications, practical applications, efficiency comparisons, ratio and regression estimators, non-sampling errors, and optimal sample design strategies. Deville and Särndal (1992) introduce calibration estimators as a method to enhance survey efficiency by incorporating auxiliary information. They adjust survey weights to match known population totals while maintaining original design weights by minimizing a distance function. This approach leads to estimators with desirable properties like unbiasedness and reduced variance, applicable to various survey designs and handling nonresponse and inconsistencies.

Supply Chain refers to the entire process of producing and delivering goods or services, from raw material sourcing to the final product reaching consumers. It involves multiple stages, including procurement, manufacturing, warehousing, transportation, and distribution. A well-optimized supply chain ensures efficiency, cost-effectiveness, and customer satisfaction. Supply Chain Management (SCM) is the strategic coordination of all supply chain activities to enhance efficiency, reduce costs, and improve service quality. It integrates planning, sourcing, production, logistics, and demand forecasting to ensure seamless operations. Supply Chain Management (SCM) consists of several key components that work together to ensure a smooth and efficient flow of goods and services. Planning involves forecasting demand and designing supply chain strategies to optimize resources. Sourcing focuses on selecting suppliers and managing procurement to ensure a steady supply of materials. Manufacturing is responsible for producing goods while maintaining high quality and efficiency standards. Logistics & Distribution play a crucial role in managing inventory, warehousing, and transportation to ensure timely deliveries. Customer service ensures that products reach customers on time, enhancing satisfaction and brand loyalty. Lastly, Returns Management handles product returns and reverse logistics, ensuring efficient recovery and recycling processes. Each of these components is essential in creating a well-structured and responsive supply chain that minimizes costs and maximizes efficiency.

Supply Chain Management with Stratified Random Sampling refers to the application of stratified random sampling techniques to improve decision-making, efficiency, and accuracy in supply chain operations. Stratified random sampling involves dividing a population (e.g., suppliers, inventory, customers, or logistics networks) into distinct subgroups or strata based on relevant characteristics, such as geographic location, product type, or supplier reliability. A random sample is then drawn from each stratum to ensure representative and unbiased data collection. By incorporating stratified random sampling in supply chain management, businesses can achieve several key benefits. Improved demand forecasting provides more accurate insights into customer preferences and regional demand variations. Enhanced supplier evaluation enables the efficient assessment of supplier performance by considering key characteristics such as production capacity and delivery reliability. Optimized inventory management ensures better stock allocation across different warehouses or distribution centers. Cost reduction minimizes waste and overstocking through more precise data analysis. Efficient quality control ensures product quality by systematically sampling from different production batches. Overall, supply chain management with stratified random sampling enhances data-driven decision-making, leading to optimized operations, reduced risks, and increased profitability.

Turner (1987) discusses the fundamentals of managing the supply chain, emphasizing coordination and efficiency across procurement, production, and distribution. He highlights the need for a systematic approach to reduce costs, improve responsiveness, and enhance overall supply chain performance. The study underscores the importance of integrating technology and strategic planning to optimize supply chain operations. Stevens (1990) highlights the importance of integrating supply chain activities to achieve efficiency and responsiveness. He emphasizes that successful supply chain management (SCM) requires coordination across procurement, production, and distribution to minimize costs and enhance customer satisfaction. The study stresses the need for a holistic approach, where all supply chain components work together as a seamless system. Ellram (1991) examines supply chain management (SCM) from an industrial organization perspective, emphasizing the strategic role of relationships among firms. She highlights the importance of collaboration, long-term partnerships, and trust in improving efficiency and competitive advantage. The study underscores how SCM extends beyond logistics to encompass broader organizational and economic factors influencing supply chain performance. Davis (1993) discusses effective supply chain management (SCM) as a key driver of competitive advantage. He highlights the need for integrating various supply chain functions, leveraging technology, and improving coordination among stakeholders. The focus is on enhancing efficiency, reducing costs, and responding swiftly to market demands through better communication and strategic planning. Cooper, *et al.* (1997) argue that supply chain management (SCM) is more than just an extension of logistics, emphasizing its strategic and integrative nature. They highlight that SCM involves cross-functional coordination, relationship management, and value creation across the entire supply chain. The study distinguishes SCM from traditional logistics by focusing on long-term collaboration, information sharing, and process integration to enhance overall efficiency and competitiveness.

Cox (1999) explores the role of power dynamics in supply chain management (SCM), emphasizing how control over resources and relationships affects value creation and distribution. He argues that effective SCM requires understanding these power structures to optimize strategic partnerships and competitive advantage. The study highlights the importance of balancing cooperation and competition to maximize overall supply chain efficiency. Mentzer *et al*. (2001) define supply chain management (SCM) as a strategic, systematic approach to coordinating business processes across firms to create value for customers. They emphasize collaboration, integration, and trust among partners for competitive advantage. SCM is categorized into direct, extended, and ultimate supply chains, highlighting the complexity of modern supply networks. Sachan and Datta (2005) review supply chain management (SCM) and logistics research, categorizing key themes such as strategy, technology, performance measurement, and integration. They highlight the evolution of SCM as an interdisciplinary field and identify research gaps, calling for more empirical studies and methodological advancements. The study emphasizes the need for a holistic approach to improve efficiency, collaboration, and innovation in supply chains. Mangan and Lalwani (2016) provide a comprehensive overview of global logistics and supply chain management (SCM), emphasizing the complexities of managing supply chains in an international context. They highlight key concepts such as globalization, technology, risk management, and sustainability. The book focuses on the strategic and operational aspects of SCM, stressing the importance of integration, efficiency, and adaptability in a rapidly evolving global market.

Singh (2003) provides an in-depth exploration of advanced sampling theory and its applications. The book covers a wide range of sampling techniques, focusing on efficient estimation methods and real-world applications. It discusses various sampling designs, auxiliary information use, and optimization strategies to improve estimation accuracy. The work is particularly useful for researchers and practitioners looking to enhance survey efficiency and precision in diverse statistical settings. Kim, *et al.* (2007) improved calibration estimation accuracy in stratified sampling by incorporating auxiliary information. They developed estimators that adjust weights within each stratum, ensuring consistency with known population totals, resulting in variance reduction and improved efficiency. Yadav and Kadilar (2013) developed an improved family of exponential estimators to estimate population mean using auxiliary variable parameters. They improved the bias and mean square error of these estimators, outperforming some exponential estimators, indicating greater efficiency in estimating the population mean. Muili *et al*. (2022) propose a modified combined ratio-type calibration estimator for stratified random sampling, addressing the issue of sensitivity to extreme values or outliers. They introduce new calibration weights to enhance robustness and efficiency. The proposed estimators outperform existing ones, exhibiting improved efficiency and reduced mean square error, making them a more reliable method for estimating the population mean in stratified sampling scenarios.

Dey *et al*. (2021) examine the impact of controllable lead time and variable demand in a smart manufacturing system within supply chain management. They highlight the role of advanced technologies and data-driven decision-making in optimizing production and inventory control. The study emphasizes how adjusting lead times and responding to fluctuating demand can enhance efficiency, reduce costs, and improve overall supply chain performance. Rouma (2022) explores the theoretical framework of supply chain uncertainties, identifying key sources of risk such as demand fluctuations, supply disruptions, and external environmental factors. The study emphasizes the importance of risk management strategies, resilience, and adaptive decision-making to mitigate uncertainties. It highlights the role of technology and collaboration in enhancing supply chain stability and performance. Tarannum and Hossain (2024) study the impact of real-time Management Information Systems (MIS) on supply chain management, highlighting its benefits in decision-making, operational efficiency, and resilience, and the technology's role in optimizing logistics. Aslam *et al.* (2024) propose a nonlinear supply chain management model that uses Takagi-Sugeno fuzzy control to optimize inventory levels, reduce delays, and improve overall supply chain efficiency by dynamically adjusting to changing conditions. Kareem *et al.* (2025) conduct a systematic literature review on the challenges of maintaining sustainable supply chains during multiple crises. They explore tensions between economic, environmental, and social sustainability, highlighting how disruptions such as pandemics, geopolitical conflicts, and climate change impact supply chain resilience. The study emphasizes the need for adaptive strategies, collaboration, and innovation to balance sustainability goals with operational efficiency in crisis-prone environments. Haval (2025) explores the use of cloud computing and data warehousing to enhance supply chain management and retail analytics. The study highlights how these technologies improve data integration, real-time decision-making, and operational efficiency. It emphasizes the role of advanced data management in optimizing inventory control, demand forecasting, and overall supply chain performance. Sanders (2025) provides a comprehensive overview of supply chain management (SCM) from a global perspective. The book explores key topics such as globalization, digital transformation, risk management, and sustainability. It emphasizes the importance of integrating technology, strategic collaboration, and data-driven decision-making to enhance supply chain efficiency and competitiveness in an increasingly interconnected world. Maghsoodi and Asadabadi (2025) propose a stratified markovian multi-preference decision support system for evaluating blockchain platforms in supply chain management, enhancing security, transparency, and efficiency, improving decision accuracy, optimizing operations, and supporting blockchain adoption in complex supply networks.

Yadav *et al.* (2023) propose generalized ratio-cum-exponential-log ratio type estimators for estimating population mean using simple random sampling. They improve estimation efficiency by incorporating auxiliary information and comparing their efficiency with existing ones. Numerical and empirical studies show the new estimators perform better under specific conditions. Yadav *et al.* (2024) propose an optimal estimation strategy for population mean in stratified random sampling, considering a linear cost function.

They develop an efficient estimator, optimize sample allocation, and compare it with existing methods, demonstrating improved estimation accuracy and cost efficiency. Zaagan *et al.* (2024) propose a cost-effective estimator for population mean in stratified random sampling, focusing on optimal sample allocation, cost reduction, and accuracy. The study compares the estimator with existing methods, demonstrating its enhancement in precision and cost efficiency. Verma *et al.* (2025) propose a new estimation strategy for stratified sampling, optimizing sample allocation to improve efficiency and cost-effectiveness. The study compares the new estimator with existing methods, demonstrating its effectiveness in accurate and cost-efficient population mean estimation.

In Daios *et al.* (2025) highlight how AI is transforming supply chain management by improving forecasting, inventory control, logistics, procurement, and risk management, which together boost efficiency, visibility, and resilience. At the same time, they note that adoption is not without hurdles such as complex implementation, uneven levels of digital maturity, and ethical concerns. Even so, AI has already shown its value in helping supply chains stay responsive during crises and disruptions. Looking ahead, the authors emphasize a shift toward more human-centric, transparent, and ethically guided AI systems that can drive supply chains to become not only more flexible and resilient but also more sustainable. The study of Kumar *et al.* (2025) explored how blockchain is revolutionizing supply chain management by ensuring transparency, security, and efficiency through tamper-proof records, traceability, and smart contracts. They show its value in procurement, logistics, and resource management, while also noting challenges like scalability, interoperability, regulations, and energy concerns. Despite these hurdles, the study highlights blockchain's growing role and its potential to integrate with AI and 5G for smarter, more resilient supply chains. Verma et al. (2025) proposed an improved strategy for estimating the population mean in stratified random sampling by combining an auxiliary attribute with an auxiliary variable. They develop a new ratio-type estimator and evaluate its performance through mean squared error (MSE) and percentage relative efficiency (PRE). Both theoretical analysis and numerical illustrations show that the estimator consistently outperforms existing methods, offering higher efficiency and practical value for applications in fields such as agriculture, healthcare, market research, and environmental studies. In Qing and Valliant (2025) proposed an extension of Cochran's classic sample-size rule, tailoring it for stratified simple random sampling to determine the minimum sample size necessary to support normal approximation based confidence intervals whether one-sided or two sided for estimating means within stratified samples.

The process of optimizing supply chain management using stratified random sampling involves defining the overall goals of the supply chain and segmenting it into distinct categories based on factors like supplier types, product categories, warehouse locations, modes of transportation, and customer demographics. We determine a sampling strategy and collect data using tools such as inventory tracking systems, IoT sensors, audits, and customer surveys. Statistical analysis is used to identify patterns, inefficiencies, and areas for improvement. Strategic decisions are made to optimize logistics, inventory levels, supplier performance, and delivery routes. The system is continuously monitored and adjusted for long-term efficiency.

The optimization process using stratified random sampling begins with defining the objective, such as minimizing sampling variance or improving the accuracy of population estimates. Once the goal is established, the next step is to stratify the population into distinct, homogeneous groups based on relevant characteristics or auxiliary variables. This stratification reduces variability within each stratum and enhances the overall precision of the sampling method. Following stratification, the process involves allocating the sample using either proportional allocation or optimal allocation techniques like Neyman allocation, which takes into account stratum variances and sizes. After determining the sample size for each stratum, the next phase is to collect data through random selection within each subgroup. Finally, the collected data is analyzed, and optimization techniques are applied to evaluate efficiency, identify improvements, and refine the sampling design for future iterations. This systematic approach ensures more efficient use of resources and enhances the reliability of statistical inferences.

2. Notations and Formulations

In this section we discuss the notations are commonly used to represent the components of the sampling process, including the population, sample, and auxiliary information. Consider a finite population of size N and random sample of size n_h is drawn using simple random sampling without replacement (SRSWOR) from the h^{th} stratum such that it is divided into L strata. Let X be an auxiliary variable and Y be the study variable taking values x_{hi} and y_{hi} respectively $(h = 1, 2, \dots, L;\ i = 1, 2, \dots, N_h)$ on i^{th} unit of the h^{th} stratum. Each stratum provides a sample of size n_h to obtain a sample of size n. With $i = 1,2, \dots N_h$ and $h = 1,2, \dots L$ let y_{hi} and x_{hi} represent the observed values of study variable Y and auxiliary variable X respectively, on the ith unit of the hth stratum. The followings may be considered as, $\bar{Y}_h$ is h^{th} stratum mean for the studied variable and X_h is h^{th} stratum mean for the auxiliary variable. The population mean of study variable is $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$ and mean of the auxiliary variable is $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$ and $W_h = \frac{N_h}{N}$ is weight of h^{th} stratum; $h = 1,2, \dots, L$. Let $\bar{y}_{st} = \bar{Y}(1 + e_0)$ and $\bar{x}_{st} = \bar{X}(1 + e_1)$, where e_0 and e_1 are errors of approximation and the value expectation of e_0 and e_1, we have

$$E(e_0) = E(e_1) = 0$$

$$E(e_0^2) = \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{yh}^2}{\bar{Y}^2}$$

$$E(e_1^2) = \sum_{h=1}^{L} W_h^2\, \lambda_h \frac{S_{xh}^2}{\bar{X}^2}$$

$$E(e_0 e_1) = \sum_{h=1}^{L} W_h^2\, \lambda_h \frac{S_{yxh}}{\bar{X}\bar{Y}}$$

where,

$$\lambda_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$$

S_{yh}^2 and S_{xh}^2represents the study variable's and auxiliary variable's in population variances, and S_{yxh}^2 represents the covariance of auxiliary and the study variables population in the h^{th}stratum.

3. Formulation of the Linear Cost Function

In stratified random sampling, the linear cost function is typically formulated to account for the total cost of sampling across different strata. It is a function of the number of units sampled from each stratum and the corresponding cost per unit. The linear cost function in stratified sampling is given by

$$C = c_0 + \sum_{h=1}^{L} c_h\, n_h \qquad (1)$$

Then, the pre-specified cost is

$$C_0 = \sum_{h=1}^{L} c_h\, n_h \qquad (2)$$

where,

$C_0 = C - c_0$ is the pre-specified cost which is fixed.

C: Total cost of the survey.

c_0: Fixed costs (e.g., setup, administration, planning).

c_h: Cost per unit sampled in stratum h.

n_h: Number of sampled units in stratum h

h: Number of stratum.; $h = 1,2, \dots, L$.

4. Some Existing Estimators in Optimization Situation

In this section we discuss the some existing estimators is as follow:

The variance of stratified random sampling is

$$t_{1(st)} = \bar{y}_{st} = \sum_{h=1}^{L} W_h \, \bar{y}_h \tag{3}$$

with MSE

$$Var\left(t_{1(st)}\right) = MSE\left(t_{1(st)}\right) = \sum_{h=1}^{L} W_h^2 \, \lambda_h S_{yh}^2$$

To minimize the objective function under budgetary limitations, the optimization constraint is formulated based on a fixed linear cost function. This ensures that the optimal solution adheres to the cost restrictions while achieving the desired efficiency

$$\begin{aligned} &Minimize \quad \sum_{h=1}^{L} \frac{W_h^2 S_{yh}^2}{n_h} \\ &\text{Subject to} \quad \sum_{h=1}^{L} c_h \, n_h \leq C_0 \\ &\qquad 2 \leq n_h \leq N_h \\ &\qquad \text{and } n_h \text{ are integer values; } h = 1,2,\dots,L. \end{aligned}$$

So that the optimum value is

$$n_h = \frac{C_0 \sqrt{\left(W_h^2 S_{yh}^2 / c_h\right)}}{\sum_{h=1}^{L} \sqrt{\left(W_h^2 S_{yh}^2 c_h\right)}}.$$

The standard ratio estimator in stratified random sampling is

$$t_{2(st)} = \bar{y}_{st} \left(\frac{\bar{X}}{\bar{x}_{st}}\right) \tag{4}$$

and MSE is

$$MSE\left(t_{2(st)}\right) = \sum_{h=1}^{L} W_h^2 \, \lambda_h \left(S_{yh}^2 + R^2 S_{xh}^2 - 2R S_{yxh}\right)$$

where, $R = \frac{\bar{Y}}{\bar{X}}$

The optimization constraint may be stated as follows in order to minimize based on the fixed linear cost function:

$$Minimize \quad \sum_{h=1}^{L} \frac{W_h^2\left(S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}\right)}{n_h}$$

Subject to $\sum_{h=1}^{L} c_h\, n_h \leq C_0$

$2 \leq n_h \leq N_h$

and n_h are integer values; $h = 1,2, \ldots, L$.

So that the optimum value is obtained as

$$n_h = \frac{C_0\sqrt{\left(W_h^2\left(S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}\right)/c_h\right)}}{\sum_{h=1}^{L}\sqrt{\left(W_h^2\left(S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}\right)c_h\right)}}.$$

Bahl and Tuteja (1991) discuss the ratio type estimator in stratified sampling and obtained MSE is

$$\boldsymbol{t}_{\mathbf{3(st)}} = \overline{\boldsymbol{y}}_{\boldsymbol{st}} exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right) \tag{5}$$

and MSE is

$$MSE\left(t_{3(st)}\right) = \sum_{h=1}^{L} W_h^2\, \lambda_h \left(S_{yh}^2 + \frac{1}{4} R^2 S_{xh}^2 - RS_{yxh}\right)$$

where, $R = \frac{\bar{Y}}{\bar{X}}$.

The optimization constraint may be stated as follows in order to minimize based on the fixed linear cost function:

$$Minimize \quad \sum_{h=1}^{L} \frac{W_h^2\left(S_{yh}^2 + \frac{1}{4} R^2 S_{xh}^2 - RS_{yxh}\right)}{n_{jh}}$$

Subject to $\sum_{h=1}^{L} C_h\, n_h \leq C_0$

$2 \leq n_h \leq N_h$

and n_h are integer values; $h = 1,2, \ldots, L$.

So that the optimum value is

$$n_h = \frac{C_0\sqrt{\left(W_h^2\left(S_{yh}^2 + \frac{1}{4}R^2S_{xh}^2 - RS_{yxh}\right)/c_h\right)}}{\sum_{h=1}^{L}\sqrt{\left(W_h^2\left(S_{yh}^2 + \frac{1}{4}R^2S_{xh}^2 - RS_{yxh}\right)c_h\right)}}.$$

Using auxiliary information in ranked set sampling (RSS), Yadav *et al.* (2019) created improved ratio estimators for population mean estimate. Their study built in multivariate stratified compromise allocation is

$$\boldsymbol{t}_{4(\mathbf{st})} = \overline{\boldsymbol{y}}_{\boldsymbol{st}}\left(\frac{ab\bar{X}+cd}{ab\,\bar{\boldsymbol{x}}_{\boldsymbol{st}}+cd}\right) \quad (6)$$

while *a, b, c,* and *d* are either fixed constants and MSE is

$$\mathrm{MSE}\left(t_{4,j(st)}\right) = \sum_{h=1}^{L} W_h^2\,\lambda_h\left(S_{yh}^2 + \theta^2R^2S_{xh}^2 - 2\theta RS_{yxh}\right)$$

where, $R = \frac{\bar{Y}}{\bar{X}}$ and $\theta = \frac{ab\bar{X}}{ab\bar{X}+\mathrm{cd}}$.

The optimization constraint may be stated as follows in order to minimize based on the fixed linear cost function:

$$Minimize \quad \sum_{h=1}^{L}\frac{W_h^2\left(S_{yh}^2 + \theta^2R^2S_{xh}^2 - 2\theta RS_{yxh}\right)}{n_h}$$

Subject to $\sum_{h=1}^{L} C_h\,n_h \le C_0$

$2 \le n_h \le N_h$

and n_{jh} takes only integer values; $h = 1,2,\dots,L$.

So that the optimum value of estimator is

$$n_h = \frac{C_0\sqrt{\left(W_h^2\left(S_{yh}^2 + \theta^2R^2S_{xh}^2 - 2\theta RS_{yxh}\right)/c_h\right)}}{\sum_{h=1}^{L}\sqrt{\left(W_h^2\left(S_{yh}^2 + \theta^2R^2S_{xh}^2 - 2\theta RS_{yxh}\right)c_h\right)}}$$

Using auxiliary information in ranked set sampling (RSS), Yadav *et al.* (2024) created improved ratio estimators for population mean estimate. Their study in stratified allocation is

$$\boldsymbol{t}_{5(\mathrm{st})} = \eta_1\, \bar{\boldsymbol{y}}_{st} + \eta_2\ \bar{\boldsymbol{y}}_{st}\left(\frac{ab\bar{X}+cd}{ab\ \bar{x}_{st}+cd}\right) \qquad (7)$$

while *a, b, c,* and *d* are either fixed constants with MSE

$$\mathrm{MSE}\left(t_{5(st)}\right) = \sum_{h=1}^{L} W_h^2\, \lambda_h Z^2$$

where,

$Z^2 = \bar{Y}^2 + \eta_1^2 a + \eta_2^2 b - 2\eta_1 + 2\eta_1\eta_2 c - 2\eta_2 d$

$a = \bar{Y}^2 + S_{yh}^2$

$b = \bar{Y}^2 + S_{yh}^2 + 3\phi^2 R^2 S_{xh}^2 - 4\phi R S_{yxh}$

$c = \bar{Y}^2 + S_{yh}^2 + \phi^2 R^2 S_{xh}^2 - 2\phi R S_{yxh}$

$d = \bar{Y}^2 + \phi^2 R^2 S_{xh}^2 - \phi R S_{yxh}$

$$\eta_1 = \frac{cd - b}{c^2 - ab}$$

$$\eta_2 = \frac{c - ad}{c^2 - ab}$$

$$R = \frac{\bar{Y}}{\bar{X}}$$

$$\phi = \frac{ab\bar{X}}{ab\bar{X} + \mathrm{cd}}$$

The optimization constraint may be stated as follows in order to minimize based on the fixed linear cost function:

$$Minimize \quad \sum_{h=1}^{L} \frac{W_h^2 Z^2}{n_h}$$

Subject to $\sum_{h=1}^{L} c_h\, n_h \le C_0$

$2 \le n_h \le N_h$

and n_{jh} takes only integer values; $h = 1,2,\dots,L$.

So that the optimum value is

$$n_h = \frac{C_0\sqrt{(W_h^2 Z^2/c_h)}}{\sum_{h=1}^{L}\sqrt{(W_h^2 Z^2 c_h)}}.$$

5. Proposed Estimator

To develop a novel approach for estimating population parameters, the original version of the proposed estimator probably combined components from several estimator types, such as product, exponential, and logarithmic estimators. The original version of the proposed estimator might be discussed in this section in the following way:

$$t_{pr(st)} = \bar{y}_{st} + K_1\bar{y}_{st}\left\{\frac{\bar{X}}{\bar{x}_{st}}\right\} + K_2\,\bar{y}_{st}\ exp\left\{\frac{(ab\bar{X}+cd)-(ab\bar{x}_{st}+cd)}{(ab\bar{X}+cd)+(ab\bar{x}_{st}+cd)}\right\} \tag{8}$$

Here, K_1 and K_2 are characterizing constants, while *a, b, c,* and *d* are either fixed constants or known parameters of the auxiliary variable. The values of K_1 and K_2 are selected in such a way that the mean squared error (MSE) of the proposed estimator is minimized. Now put the values of $\bar{y}_{st} = \bar{Y}(1+e_0)$ and $\bar{x}_{1st} = \bar{X}(1+e_2)$ in above equation and simplify, we get

$$\boldsymbol{t_{pr(st)} = \bar{Y}\Big[(1+e_0) + K_1(1+e_0-e_1-e_0e_1+e_1^2)}$$
$$\boldsymbol{+ K_2\left(1+e_0-\frac{1}{2}\theta_p e_1 - \frac{1}{2}e_0e_1 + \frac{3}{8}\theta_p^2 e_1^2\right)\Big]}$$

Subtract $\bar{Y}$ on both sides, we get

$$\boldsymbol{t_{pr(st)} - \bar{Y} = \bar{Y}\Big[(e_0) + K_1(1+e_0-e_1-e_0e_1+e_1^2)}$$
$$\boldsymbol{+ K_2\left(1+e_0-\frac{1}{2}\theta_p e_1 - \frac{1}{2}e_0e_1 + \frac{3}{8}\theta_p^2 e_1^2\right)\Big]}$$

We know that the MSE of the proposed estimator is

$$MSE\left(t_{pr(st)}\right) = E[t_{pr(st)} - \bar{Y}\,]^2$$

$$MSE\left(t_{pr(st)}\right) = \bar{Y}^2E\left[e_0 + K_1(1+e_0-e_1-e_0e_1+e_1^2) + K_2\left(1+e_0-\frac{1}{2}\theta_p e_1 - \frac{1}{2}e_0e_1 + \frac{3}{8}\theta_p^2 e_1^2\right)\right]^2$$

$$MSE\left(t_{pr(st)}\right) = \bar{Y}^2\Bigg\{e_0^2 + K_1^2(1+e_0-e_1-e_0e_1+e_1^2)^2 + K_1^2\left(1+e_0-\frac{1}{2}\theta_p e_1 - \frac{1}{2}e_0e_1 + \frac{3}{8}\theta_p^2 e_1^2\right)^2$$
$$+ 2K_1e_0(1+e_0-e_1-e_0e_1+e_1^2) + +2K_2e_0\left(1+e_0-\frac{1}{2}\theta_p e_1 - \frac{1}{2}e_0e_1 + \frac{3}{8}\theta_p^2 e_1^2\right)$$
$$+ 2K_1K_2e_0(1+e_0-e_1-e_0e_1+e_1^2)\left(1+e_0-\frac{1}{2}\theta_p e_1 - \frac{1}{2}e_0e_1 + \frac{3}{8}\theta_p^2 e_1^2\right)\Bigg\}$$

$$MSE\left(t_{pr(st)}\right) = \bar{Y}^2 E\left\{(e_0^2) + K_1^2(1 + 2e_0 - 2e_1 - 4e_0e_1 + e_0^2 + 3e_1^2)\right.$$
$$+ K_2^2\left(1 + 2e_0 - \theta_p e_1 + e_0^2 - 2\theta_p e_0 e_1 + \theta_p^2 e_1^2\right) + 2K_1(e_0 - e_0e_1 + e_0^2)$$
$$+ 2K_2\left(e_0 - \frac{1}{2}\theta_p e_0 e_1 + e_0^2\right)$$
$$\left. + 2K_1K_2\left(1 + 2e_0 - \frac{\left(\theta_p + 2\right)}{2}e_1 + e_0^2 - \left(\theta_p + 2\right)e_0e_1 + \frac{\left(7\theta_p^2 + 8\right)}{8}e_1^2\right)\right\}$$

$$MSE\left(t_{pr(st)}\right) = \bar{Y}^2\left\{E(e_0^2) + K_1^2E(1 + 2e_0 - 2e_1 - 4e_0e_1 + e_0^2 + 3e_1^2)\right.$$
$$+ K_2^2E\left(1 + 2e_0 - \theta_p e_1 + e_0^2 - 2\theta_p e_0 e_1 + \theta_p^2 e_1^2\right) + 2K_1E(e_0 - e_0e_1 + e_0^2)$$
$$+ 2K_2E\left(e_0 - \frac{1}{2}\theta_p e_0 e_1 + e_0^2\right)$$
$$\left. + 2K_1K_2E\left(1 + 2e_0 - \frac{\left(\theta_p + 2\right)}{2}e_1 + e_0^2 - \left(\theta_p + 2\right)e_0e_1 + \frac{\left(7\theta_p^2 + 8\right)}{8}e_1^2\right)\right\}$$

$$MSE\left(t_{pr(st)}\right) = \bar{Y}^2\left\{E(e_0^2) + K_1^2E\left(1 - 4E(e_0e_1) + E(e_0^2) + 3E(e_1^2)\right)\right.$$
$$+ K_2^2\left(1 + E(e_0^2) - 2\theta_p E(e_0e_1) + \theta_p^2 E(e_1^2)\right) + 2K_1\left(E(e_0^2) - E(e_0e_1)\right)$$
$$+ 2K_2\left(E(e_0^2) - \frac{1}{2}\theta_p E(e_0e_1)\right)$$
$$\left. + 2K_1K_2\left(1 + E(e_0^2) - \left(\theta_p + 2\right)E(e_0e_1) + \frac{\left(7\theta_p^2 + 8\right)}{8}E(e_1^2)\right)\right\}$$

Then obtained the MSE of proposed estimators is:

$$MSE\left(t_{pr(st)}\right) = \sum_{h=1}^{L} W_h^2\,\lambda_h M^2$$

Where,

$M^2 = \left[S_{yh}^2 + K_1^2A + K_2^2B + 2K_1C + 2K_1K_2P + 2K_2Q\right]$

$K_1 = \dfrac{PQ - BC}{AB - P^2}$

$K_2 = \dfrac{CP - AQ}{AB - P^2}$

$A = \bar{Y}^2 + S_{yh}^2 + 3\theta_p^2R^2S_{xh}^2 - 4\theta_p R S_{yxh}$

$B = \bar{Y}^2 + S_{yh}^2 - 2\theta_p R S_{yxh} + \theta_p^2 R^2 S_{xh}^2$

$C = S_{yh}^2 - R S_{yxh}$

$P = S_{yh}^2 - \frac{1}{2}\theta_p R S_{yxh}$

$Q = \bar{Y}^2 + S_{yh}^2 - (\theta_p + 2) R S_{yxh} + \frac{(7\theta_p^2 + 8)}{8} R^2 S_{xh}^2$

$\theta_p = \frac{ab\bar{X}}{ab\bar{X} + cd}.$

The optimization constraint may be stated as follows in order to minimize based on the fixed linear cost function:

$$Minimize \quad \sum_{h=1}^{L} \frac{W_h^2 M^2}{n_h}$$

$$\text{Subject to} \quad \textstyle\sum_{h=1}^{L} c_h n_h \leq C_0$$

$$2 \leq n_h \leq N_h$$

and n_{jh} takes only integer values; $h = 1,2, \ldots, L$.

So that the optimum value of the proposed estimator is

$$n_h = \frac{C_0 \sqrt{(W_h^2 M^2 / c_h)}}{\sum_{h=1}^{L} \sqrt{(W_h^2 M^2 c_h)}}.$$

6. Computational Study

We conducted a numerical analysis with based on a simulation study was conducted to estimate the average delivery time across four regions (strata) (North, South, East, and West) of India, using shipment volume as an auxiliary variable. A bivariate normal distribution was used to generate population data for each region, incorporating a correlation of 0.6 between delivery time and shipment volume. The total population consisted of 26,000 observations, stratified across the four regions (strata).

Study Variable (Y): Shipment Delivery Time.

Auxiliary Variable (X): Shipment Volume.

Let (X,Y) follow a bivariate normal distribution:

$$(X, Y) \sim N_2(\mu, \Sigma)$$

where,

$\mu = (\mu_X, \mu_X)^T$ is the mean vector

$\Sigma = \begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix}$ is the covariance matrix with σ_X^2 and σ_Y^2 being the variances of X and Y, respectively and $\rho = 0.6$ is the correlation coefficient between X and Y.

For each stratum (region r), we generate (X_r, Y_r) and $r \sim (\text{North}, \text{South}, \text{East}, \text{West})$

$$(X_r, Y_r) \sim N_2(\mu_r, \Sigma_r)$$

$\mu = \left(\mu_{X_r}, \mu_{Y_r}\right)^T$ is the mean vector and Σ_r is the region-specific covariance matrix.

Generate independent standard normal variables

$$Z_1, Z_2 \sim N(0,1)$$

$$X_r = \mu_{X_r} + \sigma_{X_r} Z_1$$

$$Y_r = \mu_{Y_r} + \rho \frac{\sigma_{X_r}}{\sigma_{Y_r}} \left(X_r - \mu_{X_r}\right) + \sigma_{Y_r} \sqrt{1 - \rho^2} Z_2$$

Total cost for delivery is C = 1500 units with an expected overhead cost c_0= 1000 units. This gives C_0 = C - c_0 = 500 units. Table 1 and Table 2 present the summary of the data.

Table 1. Summary of Data Set

Strata (h)	Population Size (N_h)	Sample Mean Delivery Time$(\bar{Y}_h)$	Sample Mean Shipment Volume $(\bar{X}_h)$	Variance of Delivery Time (S_{yh}^2)
1	5000	29.99	99.93	25.08
2	7000	28.04	120.17	36.01
3	6000	34.87	109.82	50.11
4	8000	32.03	130.27	24.65

Table 2. Summary of Data Set (Continued…)

Variance of Shipment Volume (S_{xh}^2)	Covariance (S_{yxh})	Coefficient of Skewness (β_1)	Cost per Sample (C_h)
227.69	44.59	-0.001	10
333.64	67.96	0.034	12
260.61	69.71	-0.007	11
282.40	49.22	-0.025	13

6.1. Optimization Conditions for Existing Estimators

The optimization constraints may be stated as follows in order to minimize the $MSE\left(t_{1(st)}\right)$ based on the fixed linear cost function:

$$
\begin{aligned}
&Minimize && \frac{0.92764}{8}+\frac{2.60996}{12}+\frac{2.66841}{12}+\frac{2.33352}{11} \\
&\text{Subject to} && \textstyle\sum_{h=1}^{L} c_h\, n_h \le C_0 \\
& && 2 \le n_h \le N_h \\
& && \text{and } n_h \text{ is the integer values; } h=1,2,\dots,L.
\end{aligned}
$$

The optimization constraints may be stated as follows in order to minimize the $MSE\left(t_{2(st)}\right)$ based on the fixed linear cost function:

$$
\begin{aligned}
&Minimize && \frac{0.68819}{7}+\frac{1.64704}{12}+\frac{1.73655}{12}+\frac{1.61875}{11} \\
&\text{Subject to} && \textstyle\sum_{h=1}^{L} c_h\, n_h \le C_0 \\
& && 2 \le n_h \le N_h \\
& && \text{and } n_h \text{ is the integer values; } h=1,2,\dots,L.
\end{aligned}
$$

The optimization constraints may be stated as follows in order to minimize $MSE\left(t_{3(st)}\right)$ based on the fixed linear cost function:

$$
\begin{aligned}
&Minimize && \frac{0.61832}{8}+\frac{1.79932}{11}+\frac{1.85266}{12}+\frac{1.57205}{11} \\
&\text{Subject to} && \textstyle\sum_{h=1}^{L} c_h\, n_h \le C_0 \\
& && 2 \le n_h \le N_h \\
& && \text{and } n_h \text{ is the integer values; } h=1,2,\dots,L.
\end{aligned}
$$

The optimization constraints may be stated as follows in order to minimize $MSE\left(t_{4(st)}\right)$ based on the fixed linear cost function:

$$
\begin{aligned}
&Minimize && \frac{0.68299}{8}+\frac{1.64349}{12}+\frac{1.73187}{12}+\frac{1.60973}{10} \\
&\text{Subject to} && \textstyle\sum_{h=1}^{L} c_h\, n_h \le C_0
\end{aligned}
$$

$$2 \leq n_h \leq N_h$$
and n_h is the integer values; $h = 1,2, \ldots, L$.

The optimization constraints may be stated as follows in order to minimize $MSE\left(t_{5(st)}\right)$ based on the fixed linear cost function:

$$Minimize \quad \frac{0.85783}{8} + \frac{1.24852}{12} + \frac{0.76437}{11} + \frac{1.53462}{10}$$

Subject to $\sum_{h=1}^{L} c_h\, n_h \leq C_0$

$$2 \leq n_h \leq N_h$$
and n_h is the integer values; $h = 1,2, \ldots, L$.

6.1. Optimization Conditions for Proposed Estimators is as Follow

In this section, we examine the optimization constraints may be stated as follows in order to minimize $MSE\left(t_{\mathrm{pr}(st)}\right)$ based on the fixed linear cost function:

$$Minimize \quad \frac{0.66197}{7} + \frac{0.88332}{12} + \frac{0.98948}{11} + \frac{0.01415}{10}$$

Subject to $\sum_{h=1}^{L} c_h\, n_h \leq C_0$

$$2 \leq n_h \leq N_h$$
and n_h is the integer values; $h = 1,2, \ldots, L$.

Table 3. Mean Squared Error (MSE), Percent Relative Efficiency (PRE), optimum values and the cost incurred of different estimators

Estimators	**Optimum Values (n_h)**				**MSE**	**PRE**	**Cost Incurred**
	n_1	n_2	n_3	n_4			
$t_{1(st)}$	8	12	12	11	0.76796	100.00000	499
$t_{2(st)}$	7	12	12	11	0.52744	145.60140	489
$t_{3(st)}$	8	11	12	11	0.53817	142.69840	487
$t_{4(st)}$	8	12	12	10	0.52763	145.54896	486
$t_{5(st)}$	8	12	11	10	0.43433	176.81486	475
$t_{pr(st)}$	7	12	11	10	0.25955	295.88133	465

Table 3 compares proposed and existing estimators based on Mean Squared Error (MSE), Percent Relative Efficiency (PRE), optimum values, and cost incurred. The proposed estimator has the lowest MSE and highest PRE, demonstrating superior accuracy in estimating population mean. Its optimum parameter values minimize estimation error while maintaining cost-effectiveness. The cost incurred using the method is lower or comparable to other estimators, making it practical for real-world applications like supply chain management.

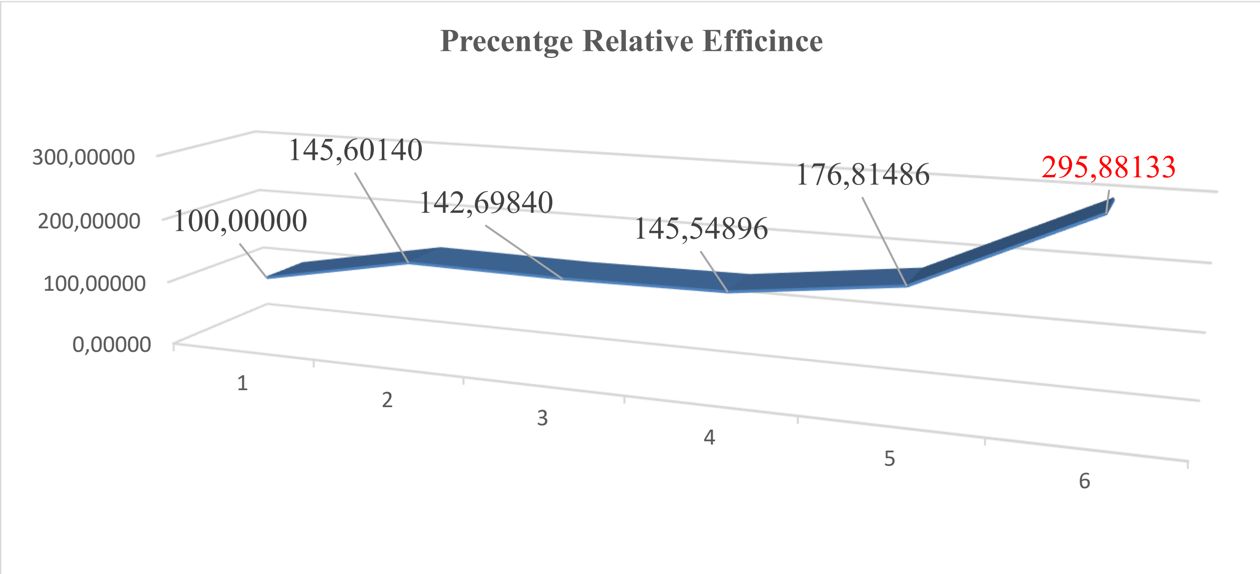

Figure 1. PRE of different estimators.

Figure 1 shows the percentage relative efficiency (PRE) of the proposed estimator as well as the existing estimators, and it is evident that the proposed estimator is more efficient compared to some of the existing estimators.

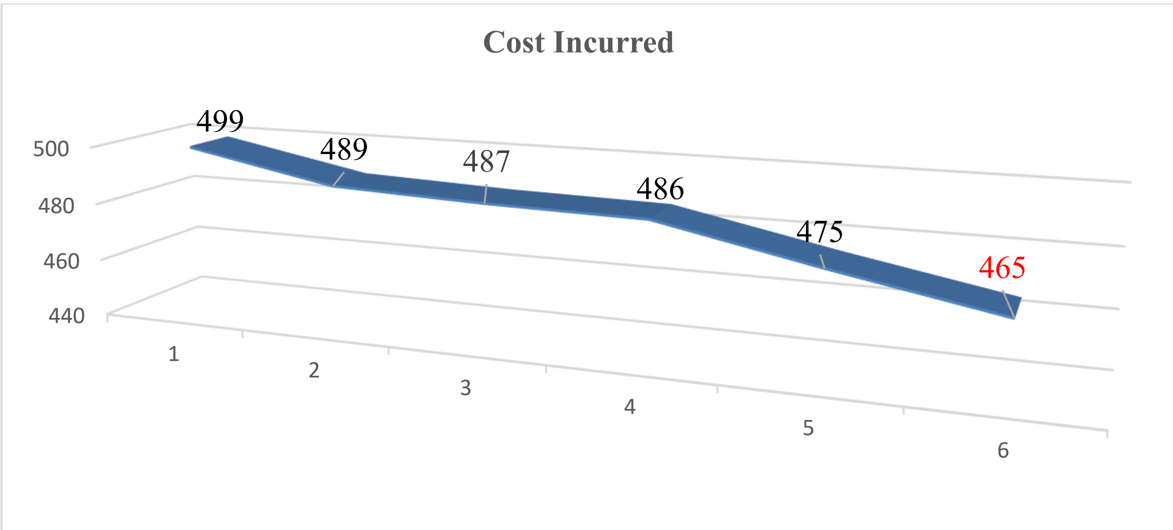

Figure 2. Cost incurred for different estimators.

Figure 2 shows that the proposed estimator is the most economical option, as it reduces costs without compromising accuracy, unlike traditional estimators that incur higher costs due to lower efficiency.

7. Results and Discussion

In this section we discuss the performance of the proposed estimator was assessed in comparison to several existing estimators using percent relative efficiency (PRE) and associated cost considerations. We examine whether the proposed estimator is more efficient than existing estimators while also incurring minimum cost. The PREs of the existing estimators are 100.0000, 145.6014, 142.6984, 145.54896, and 176.81486, whereas the PRE of the proposed estimator is 295.88133, which is significantly higher than all existing alternatives. This considerable gain in efficiency underscores the effectiveness of the proposed method in improving estimation accuracy. The associated costs for implementing each estimator were also evaluated: 499, 489, 487, 486, and 475, whereas the proposed estimator incurs a cost of 465. Interestingly, despite offering the highest PRE, the proposed estimator incurs the lowest cost among all the compared methods. This highlights the estimator’s dual advantage of superior efficiency and economic feasibility. While a common concern with advanced sampling strategies is increased implementation cost, the proposed estimator defies this expectation, thereby providing a cost-effective solution without compromising performance. The proposed estimator is not only the most efficient in terms of PRE but also the most cost-effective among the compared alternatives. These findings strongly support the utility of the proposed methodology in practical applications where both accuracy and cost are critical considerations. The results advocate for its adoption in real-world population studies, particularly where budget constraints and estimation precision are of paramount importance.

8.Conclusions

This study presents a new approach for estimating the population mean within a stratified random sampling framework, demonstrating its value in supply chain management. By incorporating an auxiliary variable shipment volume highly correlated with delivery time, the proposed method significantly enhances estimation precision while reducing operational costs. The simulation based on bivariate normal population data across four stratified regions confirms the efficiency and reliability of the estimator. This advancement supports more informed decision-making and efficient resource utilization, ultimately contributing to improved logistical performance. We conclude that the proposed estimator is more efficient and contributes significantly to the field of statistical sampling and survey methodology. This work offers improved tools for researchers and practitioners across various disciplines, enhancing the accuracy and reliability of population mean estimation. The real-life applications of the proposed method include inventory management, route optimization, demand forecasting, and cost reduction, particularly in supply chain and logistics operations.

Conflicts of Interest

The authors declare no conflict of interest.

Funding Declaration

The author declares that no funding was received for this research.

Data Availability Statement

The data generated and analyzed during this simulation study are not based on real-world data but were produced using statistical simulation methods.

Author Contributions

All authors contributed significantly to the development of this manuscript. Conceptualization: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Methodology: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Formal analysis: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Writing – original draft: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Software: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Validation: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Investigation: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Data creation: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Supervision: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Resources: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.; Writing – review & editing: VERMA, M. K., VARSHNEY, R., YADAV, S.K., GANGELE, R.K.

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