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Mathematical Modeling and Performance Optimization of Stock Preparation Unit in Paper Manufacturing Plants using GA and PSO¹

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Abstract

The prominent objective of present study is to develop an efficient mathematical model for performance optimization of stock preparation unit of paper plants using the concept of redundancy. Stock preparation in paper manufacturing involves converting raw stock into finished stock for the paper machine. This process involves several subsystems like storage tanks, repulping/slushing, deflaking, storage and mixing chests, and the paper machine itself in various redundancy strategies. For the system performance analysis, a mathematical model is developed using Markov birth death process along with reliability, availability, maintainability and dependability (RAMD) investigation of components. The Chapman-Kolmogorov differential-difference equations derived under the exponential behavior of failure and repair rates. The prediction of prominent system effectiveness measure is made using genetic algorithm and particle swarm optimization at various population sizes. Decision matrices are derived for a particular value of parameters. It is observed that predicted optimal availability of stock preparation unit is 0.9207 at a population size of 2500 after 80 iterations. It is revealed that genetic algorithm outperformed over particle swarm optimization in availability prediction of stock preparation unit. The derived results are helpful for system designers and maintenance personnel for effective decision-making for plant operations.

Keywords: Stock Preparation Unit, Steady State Availability Performance Analysis, Genetic Algorithm, Mathematical Modeling.

1. Introduction

The paper industry encompasses detailed sub processes, with stock preparation standing as a pivotal unit in which the final product's variability is shaped. Recycled paper fibers, wood, and cellulose undergo chemical and mechanical treatment before entering stock preparation, where various fiber and additive streams merge into a unified flow for the paper making machine. In pulp and paper manufacturing, important steps include raw material handling, pulp production, washing and screening, chemical/mechanical processing, bleaching, stock preparation, and the paper making process.

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The base of the stock preparation unit lies in precise constancy control with effective concern. The design of stock preparation and approach flow systems essentially affects both cost and quality of the final paper product. Sheet properties and web run ability heavily rely on the stability of the blended furnish, refined fiber quality, and the accurate mixing of fillers and chemicals. The various technologies in pulping, refining, screening, and mixing can be customized to suit specific raw material characteristics, product category requirements, and mill conditions. Enhanced run capability and high-level paper quality with minimized production costs are attainable across paper, board, and tissue machines through these improvements. Consistency serves as the bedrock of the papermaking process, with its measurement and control directly effecting product variability and costs. Even minute enhancements in consistency control over the mill can yield material savings. Eventually, the success of improvements in the stock preparation unit is indicated by increased profitability and customer attainment. Beneficially enhance the process performance, it is very important to finitely measure customer satisfaction and identify critical factors regarding products or services.

Several researchers suggested various methodologies for enhancement of reliability and performance of industrial processes. Wohl (1966) suggested methodology for system operational readiness and equipment dependability. Kumar *et al.* (1989) analyzed the availability of a washing system in the paper industry. Kumar *et al.* (1993) discussed the bleaching and screening system in the paper industry with good reduced and failed states and direct integration method is used to solve equations. Ebeling (2000) provided a comprehensive introduction of reliability and maintainability theory has been stated by Ebeling. Barabady & Kumar (2008) suggested reliability analysis of mining equipment as a study of crushing plant. Sharma & Kumar (2008) carried out the performance modeling of critical engineering systems using RAM approach. Malik and Barak (2009) conducted the economic analysis of a repairable system. Khanduja *et al.* (2009) suggested the performance analysis of the screening unit in a paper plant using Genetic Algorithm and analyzed the performance behavior of each subsystem of the screening unit using Genetic Algorithm. Iqbal and Uduman (2014) discussed the study of the stock preparation unit for paper making process and also optimized the performance of each subsystem of the stock preparation units in a paper plant by using Genetic Algorithm. Sharma and Vishwakarma (2014) used the Markov process in performance analysis of feeding system of sugar industry and emphasized the application of Markov processes. Abbas and Abdulsheeb (2016) examined an optimal path planning algorithm based on an Adaptive Multi-Objective Particle Swarm Optimization Algorithm (AMOPSO) for two case studies. Aggarwal *et al.* (2016) developed reliability, availability, maintainability & dependability (RAMD) analysis of skim milk powder production subsystem and also recognized the most critical element responsible for low production of dairy plant. Aggarwal *et al.* (2017) carried out a mathematical model for performance evaluation of the serial processes in refining system of a sugar plant using RAMD approach. Garg (2017) proposed the performance analysis of an industrial system using soft computing based hybridized technique. Tsarouhas & Besseris (2017) developed maintainability analysis in shaving blades industry. Barak *et al.* (2018) suggested a stochastic model for two-unit repairable system under priority and inspection. Barak *et al.* (2018) proposed stochastic models for various redundant systems under various redundancy strategies. Pandey *et al.* (2018) proposed reliability analysis and failure rate evaluation for critical subsystems of the dragline. Ahmadi & Amin (2019) developed an integrated chance-constrained stochastic model for a mobile phone closed-loop supply chain network with supplier selection. Choudhary *et al.* (2019) analyzed reliability, availability and maintainability to examine a cement plant. Deenadayalan and Vaishnavi (2021) innovated deep learning based on modern techniques for reliability

evaluation and forecasting using fault identification. Fasihi *et al.* (2021) proposed a bi-objective mathematical model for improvement of a fish closed-loop supply chain by using several multi-objective metaheuristic approaches. Prajapati (2022) utilized particle swarm optimization algorithm of large-scale many-objective software for architecture recovery. Saini *et al.* (2023) developed and optimized the performance of a marine power plant using metaheuristics. Kumar *et al.* (2024) conducted the performance optimization of steam turbine power plant using computational intelligence techniques.



By keeping above facts in mind, this study investigates and develop a mathematical model for performance optimization of stock preparation unit of paper plants. Stock preparation in paper manufacturing involves converting raw stock into finished stock for the paper machine. This process involves several subsystems like storage tanks, repulping/Slushing, deflaking, storage and mixing chests, and the paper machine itself in various redundancy strategies. For the system performance analysis, a mathematical model is developed using Markov birth death process along with RAMD investigation of components. The Chapman-Kolmogorov differential-difference equations derived under the exponential behavior of failure and repair rates. The prediction of prominent system effectiveness measure is made using genetic algorithm and particle swarm optimization. Decision matrices are derived for a particular value of parameters. The derived results are helpful for system designers and maintenance personnel for effective decision-making for plant operations.

2. Materials and Methods

2.1 Notations:

The mathematical model for stock preparation under is developed using the notations as appended in Table 1.

Table 1. Notations for stock preparation unit

Subsystem	Operative Mode	Failure Mode	Failure Rate μ_i	Repair Rate θ_j
Storage Tank	A	A	μ_1	θ_1
Repulping	B	B	μ_2	θ_2
Deflaking Process	C	C	μ_3	θ_3
Storage and Mixing Chest	D	D	μ_4	θ_4
Paper Machine	E	E	μ_5	θ_5
$P'_i(t)$:	Derivative of the $P_i(t)$			
$P_i(t)$	Probability that at time t the system is at i^{th} state			
C1	Represents the state in which one parallel unit is failed.			
	System is in working state with full capacity.			
	System is in failed state.			
MTBF	Mean time between failures			
MTTR	Mean time between repairs			

2.2 System description

The stock preparation is prominent subsystem in paper manufacturing plant. It comprises using five subsystems by utilizing various redundancy strategies. The configuration of subsystems shown in Figure 1. The detailed description of subsystems is as follows:

Subsystem A: This subsystem consists of a Storage Tank. It is used to hold the pulp liquids, compressed gases or mediums used for short- or long-term storage of heat or cold. The failure of this system can lead to a failure of the entire system.

Subsystem B: This subsystem consists of a Repulping or Slushing. This system helps to break down the dried primary fiber pulp or recovered paper into individual's fibers. After that the remaining flakes have to be broken down in next deflaking machinery. The failure of this subsystem may cause the complete failure of the system.

Subsystem C: This subsystem consists of a deflacking process and helps to break down small pieces (flakes) break up paper or pulp sheets into individual fibers.it helps to avoid paper quality problems, to save fiber raw material and verify enhanced operating conditions for the advancing machines in the process e.g., screening or cleaning. The failure of this system also causes the complete failure of the system.

Subsystem D: This subsystem consists of a Storage and Mixing Chest. The pulp is pumped to the storage chests or mixing chests. These chests perform as a buffer between the stock preparation and the actual paper machine. In the mixing chests, prepared stocks are mixed in proportions. Failure of this system causes the complete failure of the system.

Subsystem E: This consists of a paper machine. In this system, the final product of storage and mixing chest is send to the paper machine for producing a specific quality of paper.

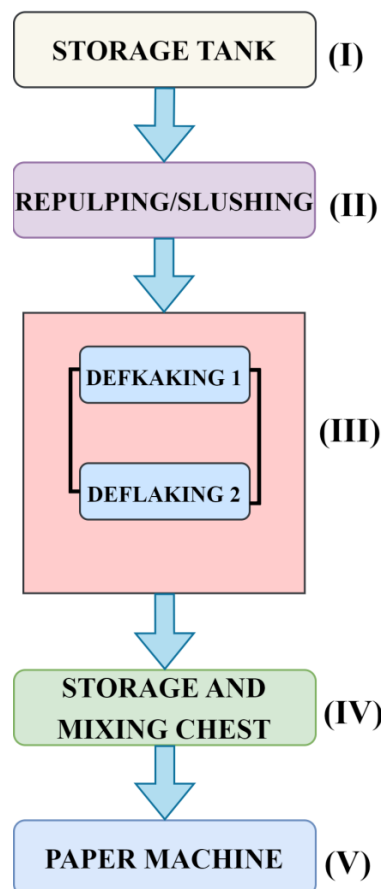


Figure 1. Flow diagram of a Stock Preparation Unit of paper plant.

2.3 Assumptions:

The model is developed under the following set of assumptions:

- Failure and repair rates are constant.
- The unit acts as new after getting repaired.
- There are no simultaneous failures among the systems.
- Sufficient repair facility available as and when needed.
- Repair and or replacement included in the service.
- Failure and repair rates follow exponential distribution.

3. RAMD Analysis

In this section, mathematical models for each subsystem are formulated using Markov methodology. The RAMD measures of each subsystem are evaluated using failure and repair rates. The failure and repair rates of each subsystem is appended in table 2.

3.1 RAMD indices for subsystem -Storage Tank:

It is a prominent component in stock preparation unit. It comprises a single component and its failure causes complete system to shut down. The failure and repair rates of storage tank are exponentially distributed. By using minimic rule, the differential-difference equations are derived (based on Figure 2) as:

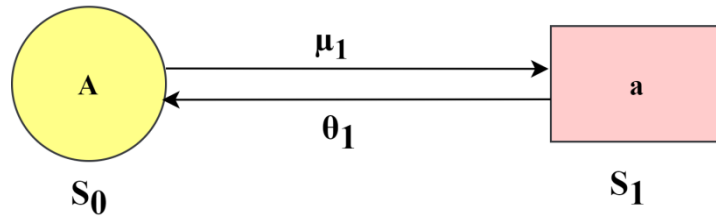


Figure 2. State transition diagram of subsystem-Storage Tank.

$$\begin{aligned}
 P_0(t + \Delta t) &= (1 - \mu_1 \Delta t)P_0(t) + \theta_1 \Delta t P_1(t) \\
 \Rightarrow P_0(t + \Delta t) &= P_0(t) - \mu_1 \Delta t P_0(t) + \theta_1 \Delta t P_1(t) \\
 \Rightarrow P_0(t + \Delta t) - P_0(t) &= \Delta t (-\mu_1 P_0(t) + \theta_1 P_1(t)) \\
 \Rightarrow \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -\mu_1 P_0(t) + \theta_1 P_1(t)
 \end{aligned}$$

Taking limit $\Delta t \rightarrow 0$, we get

$$\begin{aligned}
 \lim_{\Delta t \rightarrow 0} \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -\mu_1 P_0(t) + \theta_1 P_1(t) \\
 \lim_{t \rightarrow \infty} P_0'(t) &= -\mu_1 P_0(t) + \theta_1 P_1(t)
 \end{aligned}$$

$$0 = -\mu_1 P_0 + \theta_1 P_1 \quad (1)$$

Similarly,

$$-\theta_1 P_1 + \mu_1 P_0 = 0 \quad (2)$$

And the initial conditions are:

$$P_i(0) = \begin{cases} 1 & \text{if } i = 0 \\ 0 & \text{if } i \neq 0 \end{cases} \quad (3)$$

From equations (1-2) using (3), we get

$$\Rightarrow P_1 = \frac{\mu_1}{\theta_1} P_0 \quad (4)$$

Now using normalization condition i.e the sum of all the probabilities is equal to one $[\sum_{i=0}^1 P_i = 1]$, we get

$$\text{Availability } (A_v) = P_0 = \frac{1}{1 + \frac{\mu_1}{\theta_1}} = 0.955165692$$

Similarly,

$$\text{Reliability } (R(t)) = e^{-\mu_1 t} = e^{-0.023t}, \text{ Maintainability } (M(t)) = 1 - e^{-0.490000006t},$$

$$MTBF = \frac{1}{\mu_1} = 43.4782609h, MTTR = 2.0408163h, \text{ Dependability } (d) = 21.3043481,$$

$$D_{min} = 1 - \left(\frac{1}{d-1} \right) \left(e^{-\frac{\ln d}{d-1}} - e^{-\frac{d \ln d}{d-1}} \right) = 0.849343052$$

3.2 RAMD indices for subsystem- Repulping/Slushing:

It is a prominent component in stock preparation unit. It comprises a single component and its failure causes complete system to shut down. The failure and repair rates of repulping/slushing unit are exponentially distributed. By using minimic rule, the differential-difference equations are derived (based on Figure 3) as:

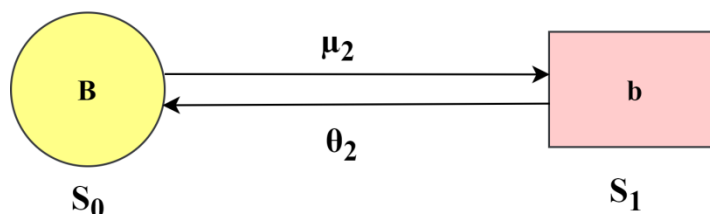


Figure 3. State transition diagram of subsystem-Repulping/Slushing.

$$P_0(t + \Delta t) = (1 - \mu_2 \Delta t) P_0(t) + \theta_2 \Delta t P_1(t)$$

$$\Rightarrow P_0(t + \Delta t) = P_0(t) - \mu_2 \Delta t P_0(t) + \theta_2 \Delta t P_1(t)$$

$$\Rightarrow P_0(t + \Delta t) - P_0(t) = \Delta t (-\mu_2 P_0(t) + \theta_2 P_1(t))$$

$$\Rightarrow \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} = -\mu_2 P_0(t) + \theta_2 P_1(t)$$

Taking limit $\Delta t \rightarrow 0$, we get

$$\lim_{\Delta t \rightarrow 0} \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} = -\mu_2 P_0(t) + \theta_2 P_1(t)$$

$$\lim_{t \rightarrow \infty} P_0' = -\mu_2 P_0(t) + \theta_2 P_1(t)$$

$$0 = -\mu_2 P_0 + \theta_2 P_1 \quad (5)$$

$$\text{Similarly, } -\theta_2 P_1 + \mu_2 P_0 = 0 \quad (6)$$

Now using normalization condition i.e the sum of all the probabilities is equal to one $[\sum_{i=0}^1 P_i = 1]$, we get

From equations (5-6) using (3), we get

$$P_0 + \frac{\mu_2}{\theta_2} P_0 = 1 \Rightarrow P_0 = \frac{1}{1 + \frac{\mu_2}{\theta_2}} \quad (7)$$

$$\therefore \text{Availability } (A_v) = \frac{1}{1 + \frac{\mu_2}{\theta_2}} = 0.974025974$$

Similarly,

$$\text{Reliability } (R(t)) = e^{-\mu_2 t} = e^{-0.02t}, \text{ Maintainability } (M(t)) = 1 - e^{-0.749999912t},$$

$$MTBF = \frac{1}{\mu_2} = 50h, MTTR = 1.33333349h, \text{ Dependability } (d) = 37.4999956,$$

$$D_{min} = 1 - \left(\frac{1}{d-1} \right) \left(e^{-\frac{\ln d}{d-1}} - e^{-\frac{d \ln d}{d-1}} \right) = 0.900702979$$

3.3 RAMD indices for subsystem-Deflaking:

It is a prominent component in stock preparation unit. It comprises two components in parallel and failure of both components resulted complete system to shut down. The failure and repair rates of deflaking unit are exponentially distributed. By using minimic rule, the differential-difference equations are derived (based on Figure 4) as:

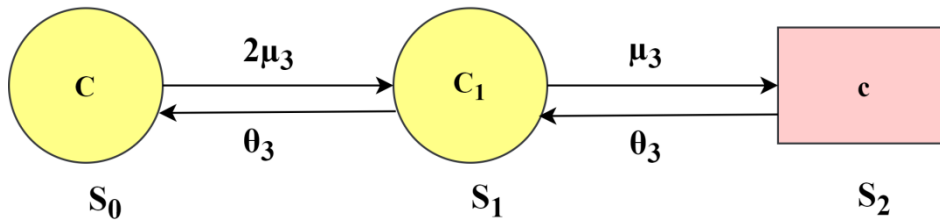


Figure 4. State transition diagram of subsystem- Deflaking.

$$\begin{aligned}
 P_0(t + \Delta t) &= (1 - 2\mu_3 \Delta t)P_0(t) + \theta_3 \Delta t P_1(t) \\
 \Rightarrow P_0(t + \Delta t) &= P_0(t) - 2\mu_3 \Delta t P_0(t) + \theta_3 \Delta t P_1(t) \\
 \Rightarrow P_0(t + \Delta t) - P_0(t) &= \Delta t (-2\mu_3 P_0(t) + \theta_3 P_1(t)) \\
 \Rightarrow \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -2\mu_3 P_0(t) + \theta_3 P_1(t)
 \end{aligned}$$

Taking limit $\Delta t \rightarrow 0$, we get

$$\begin{aligned}
 \lim_{\Delta t \rightarrow 0} \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -2\mu_3 P_0(t) + \theta_3 P_1(t) \\
 \lim_{t \rightarrow \infty} P'_0 &= -2\mu_3 P_0(t) + \theta_3 P_1(t)
 \end{aligned}$$

$$-2\mu_3 P_0 + \theta_3 P_1 = 0 \quad (8)$$

Similarly,

$$-(\mu_3 + \theta_3)P_1 + \theta_3 P_2 + 2\mu_3 P_0 = 0 \quad (9)$$

$$-\theta_3 P_2 + \mu_3 P_1 = 0 \quad (10)$$

From equations (8-10) using (3), we get

Now using normalization condition i.e the sum of all the probabilities is equal to one [$\sum_{i=0}^2 P_i = 1$], we get

$$P_1 = 2 \frac{\mu_3}{\theta_3} P_0, P_2 = 2 \frac{\mu_3^2}{\theta_3^2} P_0, P_0 = \frac{1}{1 + 2 \frac{\mu_3}{\theta_3} + 2 \frac{\mu_3^2}{\theta_3^2}} \quad (11)$$

$$\therefore \text{Availability } (A_v) = P_0 + P_1 = [1 + 2 \frac{\mu_3}{\theta_3} + 2 \frac{\mu_3^2}{\theta_3^2}]^{-1} [1 + 2 \frac{\mu_3}{\theta_3}] = 0.977301387$$

Similarly,

$$\text{Reliability } (R(t)) = e^{-\mu_3 t} = e^{-0.18t}, \text{ Maintainability } (M(t)) = 1 - e^{-7.74999988t},$$

$$MTBF = \frac{1}{\mu_3} = 5.55555556h, MTTR = 0.12903226h, \text{ Dependability } (d) = 43.0555549,$$

$$D_{min} = 1 - \left(\frac{1}{d-1} \right) \left(e^{-\frac{\ln d}{d-1}} - e^{-\frac{d \ln d}{d-1}} \right) = 0.910535213$$

3.4 RAMD indices for subsystem-Storage and Mixing Chest:

It is a prominent component in stock preparation unit. It comprises a single component and its failure causes complete system to shut down. The failure and repair rates of storage tank are exponentially distributed. By using minimic rule, the differential-difference equations are derived (based on Figure 5) as:

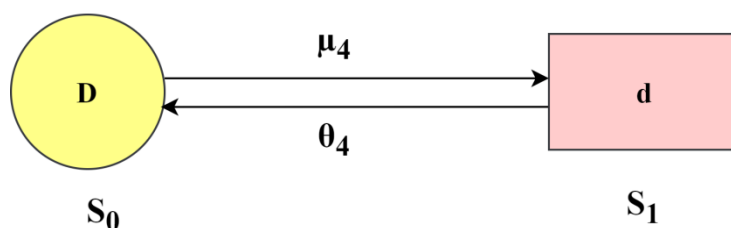


Figure 5. State transition diagram of subsystem- Storage and Mixing Chest.

$$\begin{aligned}
 P_0(t + \Delta t) &= (1 - \mu_4 \Delta t)P_0(t) + \theta_4 \Delta t P_1(t) \\
 \Rightarrow P_0(t + \Delta t) &= P_0(t) - \mu_4 \Delta t P_0(t) + \theta_4 \Delta t P_1(t) \\
 \Rightarrow P_0(t + \Delta t) - P_0(t) &= \Delta t (-\mu_4 P_0(t) + \theta_4 P_1(t)) \\
 \Rightarrow \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -\mu_4 P_0(t) + \theta_4 P_1(t)
 \end{aligned}$$

Taking limit $\Delta t \rightarrow 0$, we get

$$\begin{aligned}
 \lim_{\Delta t \rightarrow 0} \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -\mu_4 P_0(t) + \theta_4 P_1(t) \\
 \lim_{t \rightarrow \infty} P_0'(t) &= -\mu_4 P_0(t) + \theta_4 P_1(t)
 \end{aligned}$$

$$-\mu_4 P_0 + \theta_4 P_1 = 0 \quad (12)$$

Similarly,

$$-\theta_4 P_1 + \mu_4 P_0 = 0 \quad (13)$$

From equations (12-13) using (3), we get

$$P_1 = \frac{\mu_4}{\theta_4} P_0$$

Now using normalization condition i.e the sum of all the probabilities is equal to one [$\sum_{i=0}^1 P_i = 1$], we get

By solving the equation (12) and (13) and substituting the value of P_1 in equation (3), we get

$$P_0 = \frac{1}{1 + \frac{\mu_4}{\theta_4}} \quad (14)$$

$$\therefore \text{Availability } (A_v) = P_0 = \frac{1}{1 + \frac{\mu_4}{\theta_4}} = 0.9$$

Similarly,

$$\text{Reliability } (R(t)) = e^{-\mu_4 t} = e^{-0.011t}, \text{ Maintainability } (M(t)) = 1 - e^{-0.099t},$$

$$MTBF = \frac{1}{\mu_4} = 90.9090909h, MTTR = 10.1010101h, \text{ Dependability } (d) = 9,$$

$$D_{min} = 1 - \left(\frac{1}{d-1} \right) \left(e^{-\frac{\ln d}{d-1}} - e^{-\frac{d \ln d}{d-1}} \right) = 0.740069806$$

3.5 RAMD indices for subsystem-Paper Machine:

It is a prominent component in stock preparation unit. It comprises a single component and its failure causes complete system to shut down. The failure and repair rates of storage tank are exponentially distributed. By using minimic rule, the differential-difference equations are derived (based on Figure 6) as:

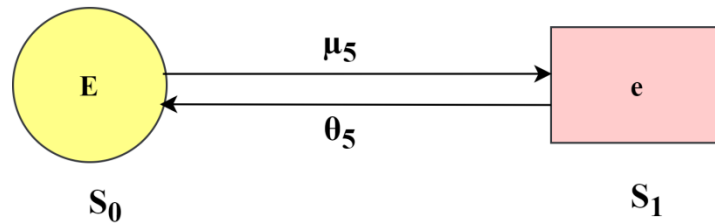


Figure 6. State transition diagram of subsystem-Paper Machine.

$$\begin{aligned}
 P_0(t + \Delta t) &= (1 - \mu_5 \Delta t)P_0(t) + \theta_5 \Delta t P_1(t) \\
 \Rightarrow P_0(t + \Delta t) &= P_0(t) - \mu_5 \Delta t P_0(t) + \theta_5 \Delta t P_1(t) \\
 \Rightarrow P_0(t + \Delta t) - P_0(t) &= \Delta t (-\mu_5 P_0(t) + \theta_5 P_1(t)) \\
 \Rightarrow \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -\mu_5 P_0(t) + \theta_5 P_1(t)
 \end{aligned}$$

Taking limit $\Delta t \rightarrow 0$, we get

$$\begin{aligned}
 \lim_{\Delta t \rightarrow 0} \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -\mu_5 P_0(t) + \theta_5 P_1(t) \\
 \lim_{t \rightarrow \infty} P_0'(t) &= -\mu_5 P_0(t) + \theta_5 P_1(t)
 \end{aligned}$$

$$-\mu_5 P_0 + \theta_5 P_1 = 0 \quad (15)$$

Similarly,

$$-\theta_5 P_1 + \mu_5 P_0 = 0 \quad (16)$$

From equations (15-16) using (3), we get

$$P_1 = \frac{\mu_5}{\theta_5} P_0$$

Now use normalization condition i.e the sum of all the probabilities is equal to one [$\sum_{i=0}^1 P_i = 1$]. By solving the equation (15) and (16) and substituting the value of P_1 in equation (3), we get

$$P_0 = \frac{1}{1 + \frac{\mu_5}{\theta_5}} \quad (17)$$

$$\therefore \text{Availability } (A_v) = P_0 = \frac{1}{1 + \frac{\mu_5}{\theta_5}} = 0.955882353$$

Similarly,

$$\begin{aligned} \text{Reliability } (R(t)) &= e^{-\mu_5 t} = e^{-0.03t}, \text{ Maintainability } (M(t)) = 1 - e^{-0.650000016t}, \\ \text{MTBF} &= \frac{1}{\mu_5} = 33.3333333h, \text{ MTTR} = 1.5384615h, \text{ Dependability } (d) = 21.6666672, \\ D_{\min} &= 1 - \left(\frac{1}{d-1} \right) \left(e^{-\frac{\ln d}{d-1}} - e^{-\frac{d \ln d}{d-1}} \right) = 0.0848268955 \end{aligned}$$

Finally, the RAMD measures of stock preparation unit are derived by using the probabilistic argument as follows:

System Reliability

$$\begin{aligned} R_{sys}(t) &= R_{ss1}(t) \times R_{ss2}(t) \times R_{ss3}(t) \times R_{ss4}(t) \times R_{ss5}(t) \\ \Rightarrow R_{sys}(t) &= e^{-0.023t} \times e^{-0.02t} \times e^{-0.18t} \times e^{-0.011t} \times e^{-0.03t} = e^{-0.264t} \end{aligned}$$

System Availability

$$\begin{aligned} A_{sys}(t) &= A_{ss1} \times A_{ss2} \times A_{ss3} \times A_{ss4} \times A_{ss5} \\ &= 0.955165692 \times 0.974025974 \times 0.977301387 \times 0.9 \times 0.955882353 = 0.782212446 \end{aligned}$$

System Maintainability

$$\begin{aligned} M_{sys}(t) &= M_{ss1}(t) \times M_{ss2}(t) \times M_{ss3}(t) \times M_{ss4}(t) \times M_{ss5}(t) \\ \Rightarrow (1 - e^{-0.490000006t}) \times (1 - e^{-0.749999912t}) \times (1 - e^{-7.74999988t}) \times (1 - e^{-0.099t}) \times (1 - e^{-0.650000016t}) &= (1 - e^{-9.73899981t}) \end{aligned}$$

System Dependability

$$\begin{aligned} D_{\min(sys)}(t) &= D_{\min(ss1)} \times D_{\min(ss2)} \times D_{\min(ss3)} \times D_{\min(ss4)} \times D_{\min(ss5)} \\ &= 0.849343052 \times 0.900702979 \times 0.910535213 \times 0.740069806 \times 0.0848268955 = 0.437288184 \end{aligned}$$

Performance modeling and Optimization of Stock Preparation Unit

In this section, a mathematical model of stock preparation unit is developed by using Markov birth death process and Chapman-Kolmogorov differential-difference equations derived by considering constant failure and repair rates. The steady state availability expression is derived from derived mathematical model and treated as the objective function for availability optimization. All the failure and repair rates are considered as the decision variables. The nature inspired algorithms genetic algorithm (GA) and particle swarm optimization (PSO) are utilized for optimization of objective function. The mathematical model is developed based on state transition diagram (Figure 7) by using simple probabilistic arguments as follows:

$$\begin{aligned} P_0(t + \Delta t) &= (1 - \mu_1 \Delta t - \mu_2 \Delta t - 2\mu_3 \Delta t - \mu_4 \Delta t + \mu_5 \Delta t)P_0(t) + \theta_1 \Delta t P_7(t) + \theta_2 \Delta t P_8(t) \\ &\quad + \theta_3 \Delta t P_1(t) + \theta_4 \Delta t P_9(t) + \theta_5 \Delta t P_{10}(t) \end{aligned}$$

$$\begin{aligned} \Rightarrow P_0(t + \Delta t) &= P_0(t) - \mu_1 \Delta t P_0(t) - \mu_2 \Delta t P_0(t) - 2\mu_3 \Delta t P_0(t) - \mu_4 \Delta t P_0(t) + \mu_5 \Delta t P_0(t) \\ &\quad + \theta_1 \Delta t P_7(t) + \theta_2 \Delta t P_8(t) + \theta_3 \Delta t P_1(t) + \theta_4 \Delta t P_9(t) + \theta_5 \Delta t P_{10}(t) \\ \Rightarrow P_0(t + \Delta t) - P_0(t) &= \Delta t (-\mu_1 P_0(t) - \mu_2 P_0(t) - 2\mu_3 P_0(t) - \mu_4 P_0(t) + \mu_5 P_0(t) + \theta_1 P_7(t) + \theta_2 P_8(t) \\ &\quad + \theta_3 P_1(t) + \theta_4 P_9(t) + \theta_5 P_{10}(t)) \end{aligned}$$

Taking $\lim_{\Delta t \rightarrow 0}$, we get

$$\begin{aligned} \lim_{\Delta t \rightarrow 0} \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} &= -(\mu_1 + \mu_2 + 2\mu_3 + \mu_4 + \mu_5)P_0(t) + \theta_1 P_7(t) + \theta_2 P_8(t) + \theta_3 P_1(t) + \theta_4 P_9(t) \\ &\quad + \theta_5 P_{10}(t) \\ \Rightarrow P'_0(t) &= -(\mu_1 + \mu_2 + 2\mu_3 + \mu_4 + \mu_5)P_0(t) + \theta_1 P_7(t) + \theta_2 P_8(t) + \theta_3 P_1(t) + \theta_4 P_9(t) \\ &\quad + \theta_5 P_{10}(t) \end{aligned}$$

Now, applying $\lim_{t \rightarrow \infty}$, we obtain

$$\begin{aligned} \Rightarrow \lim_{t \rightarrow \infty} P'_0(t) &= -(\mu_1 + \mu_2 + 2\mu_3 + \mu_4 + \mu_5)P_0(t) + \theta_1 P_7(t) + \theta_2 P_8(t) + \theta_3 P_1(t) + \theta_4 P_9(t) \\ &\quad + \theta_5 P_{10}(t) \\ \Rightarrow (\mu_1 + \mu_2 + 2\mu_3 + \mu_4 + \mu_5)P_0 &= \theta_1 P_7 + \theta_2 P_8 + \theta_3 P_1 + \theta_4 P_9 + \theta_5 P_{10} \end{aligned} \quad (18)$$

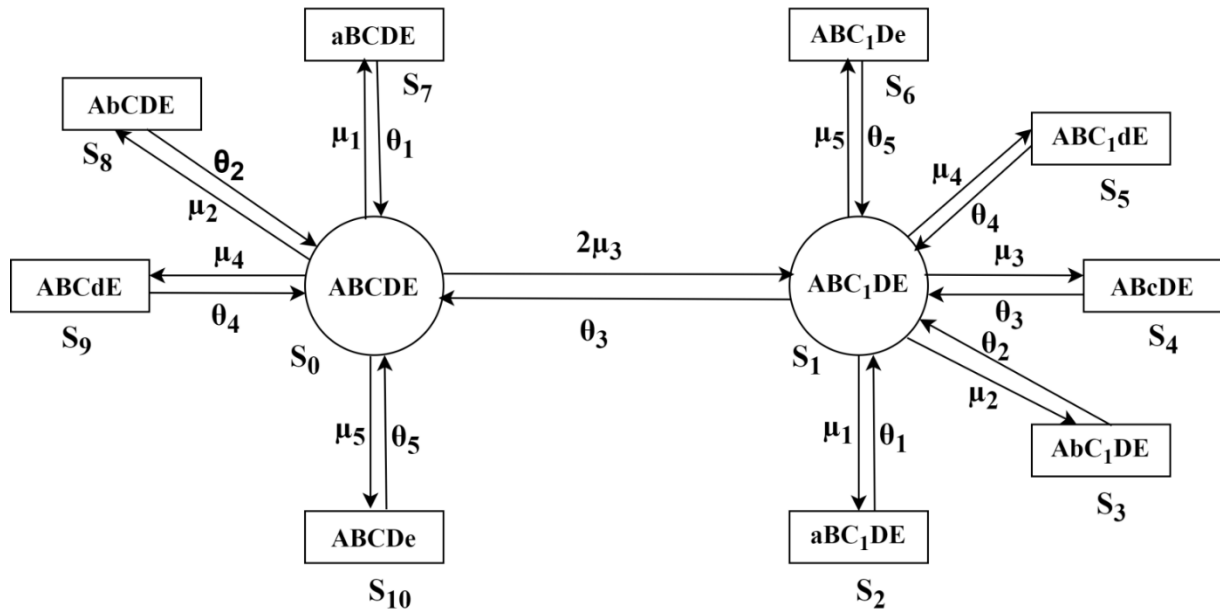


Figure 7. State transition diagram of stock preparation unit of paper plant.

Similarly,

$$(\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \theta_3)P_1 = \theta_1 P_2 + \theta_2 P_3 + \theta_3 P_4 + \theta_4 P_5 + \theta_5 P_6 + 2\mu_3 P_0 \quad (19)$$

$$\theta_1 P_2 = \mu_1 P_1 \quad (20)$$

$$\theta_2 P_3 = \mu_2 P_1 \quad (21)$$

$$\theta_3 P_4 = \mu_3 P_1 \quad (22)$$

$$\theta_4 P_5 = \mu_4 P_1 \quad (23)$$

$$\theta_5 P_6 = \mu_5 P_1 \quad (24)$$

$$\theta_1 P_7 = \mu_1 P_0 \quad (25)$$

$$\theta_2 P_8 = \mu_2 P_0 \quad (26)$$

$$\theta_4 P_9 = \mu_4 P_0 \quad (27)$$

$$\theta_5 P_{10} = \mu_5 P_0 \quad (28)$$

The normalization equation is:

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} = 1$$

By using above system of equations (18-28) and initial conditions (3), we get

$$\begin{aligned} &\Rightarrow P_0 + 2 \frac{\mu_3}{\theta_3} P_0 + \frac{\mu_1}{\theta_1} 2 \frac{\mu_3}{\theta_3} P_0 + \frac{\mu_2}{\theta_2} 2 \frac{\mu_3}{\theta_3} P_0 + \frac{\mu_3}{\theta_3} 2 \frac{\mu_3}{\theta_3} P_0 + \frac{\mu_4}{\theta_4} 2 \frac{\mu_3}{\theta_3} P_0 + \frac{\mu_5}{\theta_5} 2 \frac{\mu_3}{\theta_3} P_0 + \frac{\mu_1}{\theta_1} P_0 + \frac{\mu_2}{\theta_2} P_0 + \\ &\frac{\mu_4}{\theta_4} P_0 + \frac{\mu_5}{\theta_5} P_0 = 1 \\ &\Rightarrow P_0 \left(1 + 2 \frac{\mu_3}{\theta_3} P_0 + 2 \frac{\mu_1 \mu_3}{\theta_1 \theta_3} + 2 \frac{\mu_2 \mu_3}{\theta_2 \theta_3} + 2 \frac{\mu_3 \mu_3}{\theta_3 \theta_3} + 2 \frac{\mu_4 \mu_3}{\theta_4 \theta_3} + 2 \frac{\mu_5 \mu_3}{\theta_5 \theta_3} + \frac{\mu_1}{\theta_1} + \frac{\mu_2}{\theta_2} + \frac{\mu_4}{\theta_4} + \frac{\mu_5}{\theta_5} \right) = 1 \\ &\Rightarrow P_0 = \frac{1}{1 + 2 \frac{\mu_3}{\theta_3} + 2 \frac{\mu_1 \mu_3}{\theta_1 \theta_3} + 2 \frac{\mu_2 \mu_3}{\theta_2 \theta_3} + 2 \frac{\mu_3 \mu_3}{\theta_3 \theta_3} + 2 \frac{\mu_4 \mu_3}{\theta_4 \theta_3} + 2 \frac{\mu_5 \mu_3}{\theta_5 \theta_3} + \frac{\mu_1}{\theta_1} + \frac{\mu_2}{\theta_2} + \frac{\mu_4}{\theta_4} + \frac{\mu_5}{\theta_5}} \end{aligned}$$

$$\text{Steady State Availability (Av)} = P_0 + P_1 \quad (29)$$

$$\text{Objective Function Av} = P_0 + 2 \frac{\mu_3}{\theta_3} P_0 = P_0 \left(1 + 2 \frac{\mu_3}{\theta_3} \right) =$$

$$\left(\frac{1}{1 + 2 \frac{\mu_3}{\theta_3} + 2 \frac{\mu_1 \mu_3}{\theta_1 \theta_3} + 2 \frac{\mu_2 \mu_3}{\theta_2 \theta_3} + 2 \frac{\mu_3 \mu_3}{\theta_3 \theta_3} + 2 \frac{\mu_4 \mu_3}{\theta_4 \theta_3} + 2 \frac{\mu_5 \mu_3}{\theta_5 \theta_3} + \frac{\mu_1}{\theta_1} + \frac{\mu_2}{\theta_2} + \frac{\mu_4}{\theta_4} + \frac{\mu_5}{\theta_5}} \right) \left(1 + 2 \frac{\mu_3}{\theta_3} \right) \quad (30)$$

4. Numerical Results & Discussion:

In this section, numerical results for system effectiveness measures are derived for a particular case in three phases. The values of the decision variables are appended in table 2.

Table 2. Failure and repair rates of subsystems of stock preparation unit of paper plant

Sr. No.	Sub-system	Failure-rate (μ_i)	Repair-rate (θ_j)
1	Storage Tank (I)	$\mu_1=0.023$	$\theta_1=0.49$
2	Repulping/Slushing (II)	$\mu_2=0.02$	$\theta_2=0.75$
3	Deflaking Process (III)	$\mu_3=0.06$	$\theta_3=0.5$
4	Storage and Mixing Chest (IV)	$\mu_4=0.011$	$\theta_4=0.099$
5	Paper Machine (V)	$\mu_5=0.03$	$\theta_5=0.65$

From Table 3, it is observed that reliability and availability of the stock preparation unit is less in comparison to its weakest subsystem. The maximum MTBF is 90.9090 and minimum is 5.5556 associated with subsystems storage & Mixing chest and deflaking process respectively. The MTBF for the stock preparation unit is 223.276241.

Similarly, MTTR of the stock preparation unit is 15.1426537.

Table 3. RAMD indices for subsystems and system of the Stock Preparation unit

RAMD indices of subsystems	SSI	SSII	SSIII	SSIV	SSV	System
Reliability	$e^{-0.023t}$	$e^{-0.02t}$	$e^{-0.18t}$	$e^{-0.011t}$	$e^{-0.03t}$	$e^{-0.264t}$
Availability	0.955165692	0.974025974	0.977301387	0.9	0.955882353	0.782212446
Maintainability	$(1 - e^{-0.490000006t})$	$(1 - e^{-0.749999912t})$	$(1 - e^{-7.74999988t})$	$(1 - e^{-0.099t})$	$(1 - e^{-0.650000016t})$	$(1 - e^{-9.73899981t})$
Dependability (Dmin)	0.849343052	0.900702979	0.910535213	0.740069806	0.848268955	0.437288184
MTBF (hrs.)	43.4782609	50	5.55555556	90.9090909	33.3333333	223.276241
MTTR (hrs.)	2.0408163	1.33333349	0.12903226	10.1010101	1.5384615	15.1426537
Dependability ratio (d)	21.3043481	37.4999956	43.0555549	9	21.6666672	

The effect of variation in failure rates of various subsystems analyzed on reliability of subsystems and stock preparation unit and appended in tables 4, 5, 6,7 and 8. It is revealed that with the increment of failure rate of subsystem there is a steep decline in the reliability of subsystem and stock preparation unit. It is observed that subsystem namely Deflaking is the least reliable among all subsystems having reliability 0.22313.

Table 4. Effect of change in failure rate (μ_1) on the reliability of the Storage Tank and stock preparation unit

Time (hrs.)	System				Subsystem I			
	$\mu_1 = 0.013$	$\mu_1 = 0.023$	$\mu_1 = 0.033$	$\mu_1 = 0.043$	$\mu_1 = 0.013$	$\mu_1 = 0.023$	$\mu_1 = 0.033$	$\mu_1 = 0.043$
10	0.078866	0.794534	0.718924	0.650509	0.878095	0.794534	0.718924	0.650509
20	0.00622	0.631284	0.516851	0.423162	0.771052	0.631284	0.516851	0.423162
30	0.000491	0.501576	0.371577	0.275271	0.677057	0.501576	0.371577	0.275271
40	3.87E-05	0.398519	0.267135	0.179066	0.594521	0.398519	0.267135	0.179066
50	3.05E-06	0.316637	0.19205	0.116484	0.522046	0.316637	0.19205	0.116484

Table 5. Effect of change in failure rate (μ_2) on the reliability of the Repulping/Slushing and stock preparation unit

Time (hrs.)	System				Subsystem II			
	$\mu_2 = 0.01$	$\mu_2 = 0.02$	$\mu_2 = 0.03$	$\mu_2 = 0.04$	$\mu_2 = 0.01$	$\mu_2 = 0.02$	$\mu_2 = 0.03$	$\mu_2 = 0.04$
10	0.078866	0.071361	0.06457	0.058426	0.904837	0.818731	0.740818	0.67032
20	0.00622	0.005092	0.004169	0.003414	0.818731	0.67032	0.548812	0.449329
30	0.000491	0.000363	0.000269	0.000199	0.740818	0.548812	0.40657	0.301194
40	3.87E-05	2.59E-05	1.74E-05	1.17E-05	0.67032	0.449329	0.301194	0.201897
50	3.05E-06	1.85E-06	1.12E-06	6.81E-07	0.606531	0.367879	0.22313	0.135335

Table 6. Effect of change in failure rate (μ_3) on the reliability of the Deflaking and stock preparation unit

Time (hrs.)	System				Subsystem III			
	$\mu_3 = 0.05$	$\mu_3 = 0.06$	$\mu_3 = 0.07$	$\mu_3 = 0.08$	$\mu_3 = 0.05$	$\mu_3 = 0.06$	$\mu_3 = 0.07$	$\mu_3 = 0.08$
10	0.096328	0.071361	0.066537	0.039164	0.22313	0.165299	0.122456	0.090718
20	0.009279	0.005092	0.004427	0.001534	0.049787	0.027324	0.014996	0.00823
30	0.000894	0.000363	0.000295	6.01E-05	0.011109	0.004517	0.001836	0.000747
40	8.61E-05	2.59E-05	1.96E-05	2.35E-06	0.002479	0.000747	0.000225	6.77E-05
50	8.29E-06	1.85E-06	1.3E-06	9.21E-08	0.000553	0.000123	2.75E-05	6.14E-06

Table 7. Effect of change in failure rate (μ_4) on the reliability of the Storage and Mixing Chest and stock preparation unit

Time (hrs.)	System				Subsystem IV			
	$\mu_4 = 0.001$	$\mu_4 = 0.011$	$\mu_4 = 0.021$	$\mu_4 = 0.031$	$\mu_4 = 0.001$	$\mu_4 = 0.011$	$\mu_4 = 0.021$	$\mu_4 = 0.031$
10	0.078866	0.071361	0.06457	0.058426	0.99005	0.895834	0.810584	0.733447
20	0.00622	0.005092	0.004169	0.003414	0.980199	0.802519	0.657047	0.537944
30	0.000491	0.000363	0.000269	0.000199	0.970446	0.718924	0.532592	0.394554
40	3.87E-05	2.59E-05	1.74E-05	1.17E-05	0.960789	0.644036	0.431711	0.289384
50	3.05E-06	1.85E-06	1.12E-06	6.81E-07	0.951229	0.57695	0.349938	0.212248

Table 8. Effect of change in failure rate (μ_5) on the reliability of the Paper Machine and stock preparation unit

Time (hrs.)	System				Subsystem V			
	$\mu_5 = 0.02$	$\mu_5 = 0.03$	$\mu_5 = 0.04$	$\mu_5 = 0.05$	$\mu_5 = 0.02$	$\mu_5 = 0.03$	$\mu_5 = 0.04$	$\mu_5 = 0.05$
10	0.078866	0.071361	0.06457	0.058426	0.99005	0.895834	0.810584	0.733447
20	0.00622	0.005092	0.004169	0.003414	0.980199	0.802519	0.657047	0.537944
30	0.000491	0.000363	0.000269	0.000199	0.970446	0.718924	0.532592	0.394554
40	3.87E-05	2.59E-05	1.74E-05	1.17E-05	0.960789	0.644036	0.431711	0.289384
50	3.05E-06	1.85E-06	1.12E-06	6.81E-07	0.951229	0.57695	0.349938	0.212248

From Table 9, it is identified that with the passes of time the probability of restoration to operative state of any subsystem and stock preparation unit increases. It is revealed that any failure in subsystem Deflaking takes maximum time in its restoration.

Table 9. Variation of maintainability of subsystems with time

Time (hrs.)	MSSI	MSSII	MSSIII	MSSIV	MSSV
10	0.992553	0.999447	1.000000	0.628423	0.998497
20	0.999945	1.000000	1.000000	0.861931	0.999998
30	1.000000	1.000000	1.000000	0.948697	1.000000
40	1.000000	1.000000	1.000000	0.980937	1.000000
50	1.000000	1.000000	1.000000	0.992917	1.000000

In second phase, the steady state availability of stock preparation unit is evaluated for a particular set of parametric values. The impact of variation in failure and repair rates on steady state availability is explored. It is identified that steady state availability sharply declines with the 20% variation in various failure rates and shows the significant enhancement with the increment of repair rate of the subsystems.

Table 10. Effect of Failure rates on the availability after $\mu_i+20\%$ variation

μ_1	Base Values	$\mu_2+20\%$ of μ_2	$\mu_3+20\%$ of μ_3	$\mu_4+20\%$ of μ_4	$\mu_5+20\%$ of μ_5
0.023	0.873077	0.871927	0.86326	0.869449	0.866097
0.024	0.871524	0.870378	0.861742	0.867909	0.864569
0.025	0.869977	0.868835	0.860229	0.866375	0.863046
0.026	0.868435	0.867297	0.858722	0.864845	0.861528
0.027	0.866898	0.865765	0.857219	0.863322	0.860016
0.028	0.865367	0.864238	0.855722	0.861803	0.85851
0.029	0.863842	0.862716	0.854231	0.86029	0.857008

Table 11. Effect of Repair rates on the availability after $\theta_i+20\%$ variation

θ_1	Base Values	$\theta_2+10\%$ of θ_2	$\theta_3+10\%$ of θ_3	$\theta_4+10\%$ of θ_4	$\theta_5+10\%$ of θ_5
0.49	0.873295	0.87351	0.874471	0.88135	0.876559
0.491	0.873367	0.873077	0.873295	0.873295	0.873295
0.492	0.873438	0.87315	0.873367	0.873367	0.873367
0.493	0.87351	0.873222	0.873438	0.873438	0.873438
0.494	0.873077	0.874037	0.880909	0.876123	0.87898
0.495	0.87315	0.87411	0.880983	0.876196	0.879054
0.496	0.873222	0.874183	0.881057	0.87627	0.879127

In the last phase, nature inspired algorithms GA and PSO applied on objective function equation (30) to predict the availability of the stock preparation unit and estimate the best fitted parameters value. Table 12 -13 appended the optimization parameters and search space for prediction the availability.

Table 12. Different parameters used in optimization techniques

Algorithm	Parameters
Genetic Algorithm	Population size: 500, 1000, 1500, 2000, 2500 Number of maximum iterations: 50, 60, 70, 80, 90; Crossover rate:0.8; Mutation rate: 0.9
Particle Swarm Optimization	Population size: 500, 1000, 1500, 2000, 2500 Number of maximum iterations; 50, 60, 70, 80, 90 Inertia weight: 0.99, Damping ratio: 0.8, Global best: 2.684, Personal best: 1.789

Table 13. Search space of failure and repair rates

Sr. No.	Name of subsystem	Range of failure rates (μ_i)	Range of repair rates (θ_j)
1	Storage Tank (I)	[0.02, 0.09]	[0.10, 0.50]
2	Repulping/slushing (II)	[0.01, 0.09]	[0.05, 0.08]
3	Deflacking (III)	[0.02, 0.08]	[0.10, 0.90]
4	Storage and Mixing Chest (IV)	[0.01, 0.07]	[0.02, 0.10]
5	Paper Machine(V)	[0.02, 0.06]	[0.10, 0.55]

Table 14. Availability of Stock Preparation Unit in a Paper Plant using Genetic Algorithm

Pop.\Iter	50	60	70	80	90
500	0.9038	0.8817	0.8877	0.9085	0.8856
1000	0.8996	0.9098	0.9026	0.8933	0.9128
1500	0.9119	0.9117	0.9140	0.9054	0.8989
2000	0.9077	0.9153	0.8967	0.9038	0.9063
2500	0.9107	0.8944	0.9078	0.9207	0.8996

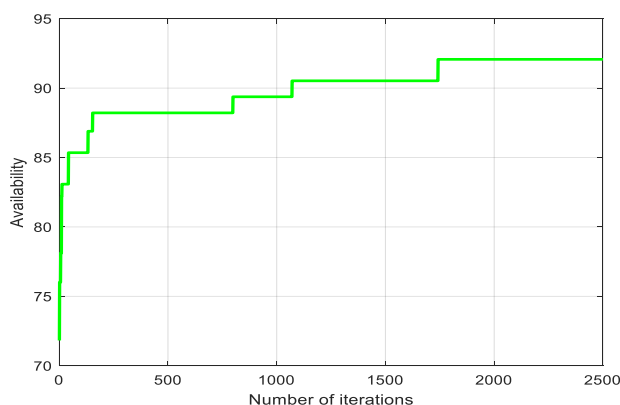

Figure 8. Effect of Number of Iteration on Fitness (Stock Preparation Unit Availability) at population 2500 and iteration 80.

Figure 8 represents the convergence of availability at iteration 80 with population size of 2500. Table 14 displays optimized availability achieved through Genetic Algorithm (GA) with different iterations and population sizes. The system reaches its peak availability of 0.9207 with a population size of 2500 and 80 iterations. Table 15 appends parameter values corresponding to different population sizes and iterations.

Table 15. Best fitted parameter values of various failure and repair rates of Stock Preparation Unit in a paper plant at different population with different iterations by GA

Pop.\ Iter		50	60	70	80	90
500	μ_1	0.0504	0.0060	0.0177	0.0206	0.0179
	μ_2	0.0365	0.0046	0.0063	0.0302	0.0046
	μ_3	0.0015	0.01093	0.0669	0.0356	0.0026
	μ_4	0.0031	0.0038	0.0118	0.0181	0.0034
	μ_5	0.0236	0.0032	0.0773	0.0127	0.1142
	θ_1	3.4943	1.4621	1.3007	2.1941	2.0248
	θ_2	1.4973	0.3487	1.1370	1.1207	2.4282
	θ_3	0.7854	0.4702	0.8720	0.5831	0.6475
	θ_4	0.1336	0.2562	0.2281	0.2982	0.0010
	θ_5	1.8900	2.3811	1.2619	3.5605	1.7821
1000	μ_1	0.0012	0.0011	0.0265	0.0067	0.0408
	μ_2	0.0013	0.0200	0.0393	0.0166	0.0019
	μ_3	0.0045	0.0032	0.0978	0.1068	0.0069
	μ_4	0.0177	0.0048	0.0329	0.0046	0.0038
	μ_5	0.0179	0.0051	0.0006	0.0042	0.0239
	θ_1	1.4609	1.0949	3.3332	1.2715	3.5903
	θ_2	1.7594	2.2944	4.2528	3.2362	2.2825
	θ_3	1.8803	1.1983	1.2508	1.0124	0.7723
	θ_4	0.0487	0.2300	0.1932	0.1452	0.0735
	θ_5	1.7157	1.7721	0.6823	0.9426	1.9739
1500	μ_1	0.0042	0.0180	0.0046	0.0034	0.0180
	μ_2	0.0094	0.0220	0.0011	0.0251	0.0022
	μ_3	0.0608	0.0617	0.0010	0.0252	0.0229
	μ_4	0.0031	0.0014	0.0085	0.0030	0.0133
	μ_5	0.0422	0.0674	0.0110	0.0059	0.0475
	θ_1	2.1824	1.3717	0.6944	1.7799	1.7582
	θ_2	0.8122	1.6712	1.0194	0.8368	0.2901
	θ_3	1.5597	1.1024	0.6315	1.9500	1.8227
	θ_4	0.4358	0.4485	0.7802	0.1653	0.1790
	θ_5	1.0447	1.5173	5.8090	2.0613	2.2694
2000	μ_1	0.0044	0.0208	0.0136	0.0005	0.0099
	μ_2	0.0037	0.0102	0.0232	0.0464	0.0423
	μ_3	0.0124	0.0406	0.1503	0.0439	0.0346
	μ_4	0.0034	0.0145	0.0054	0.0021	0.0151
	μ_5	0.0490	0.0098	0.0091	0.0667	0.0115
	θ_1	0.8209	2.3613	0.7368	1.4062	1.7682
	θ_2	3.3630	0.5976	1.0014	6.0609	1.0181
	θ_3	2.9321	1.3748	1.6188	0.8981	1.0237
	θ_4	0.0995	0.5213	0.3686	0.0666	0.2828
	θ_5	1.9543	1.1836	2.3837	1.5564	2.4916
2500	μ_1	0.0266	0.0075	0.0196	0.0238	0.0061
	μ_2	0.0049	0.0024	0.0094	0.0084	0.0111
	μ_3	0.0125	0.0276	0.0111	0.0354	0.0487
	μ_4	0.0021	0.0082	0.0225	0.0007	0.0101
	μ_5	0.0585	0.0083	0.0065	0.0012	0.0039
	θ_1	1.3321	2.1789	1.4795	5.9190	1.4313
	θ_2	1.1499	0.5103	1.2239	3.3782	1.2693
	θ_3	0.4823	0.8508	0.7346	0.4195	0.6056
	θ_4	0.3527	1.1412	0.3640	0.4086	0.6741
	θ_5	4.6458	2.0315	1.8428	1.0030	0.8311

The availability measures are presented in Table 16, while Figure 9 shows the convergence of at population 1000 and at 60 iterations. The system converges to achieve its peak availability of 0.8947. Further details of parameter values for various population sizes and iterations are provided in Table 17. It is observed that in availability prediction of stock preparation unit GA outperforms over PSO.

Table 16. Availability of Stock Preparation Unit in a Paper Plant to various population sizes after several no. of iterations Using PSO

Pop.\Iter	50	60	70	80	90
500	0.8947	0.8947	0.8947	0.8947	0.8947
1000	0.8947	0.8947	0.8947	0.8947	0.8947
1500	0.8947	0.8947	0.8947	0.8947	0.8947
2000	0.8947	0.8947	0.8947	0.8947	0.8947
2500	0.8947	0.8947	0.8947	0.8947	0.8947

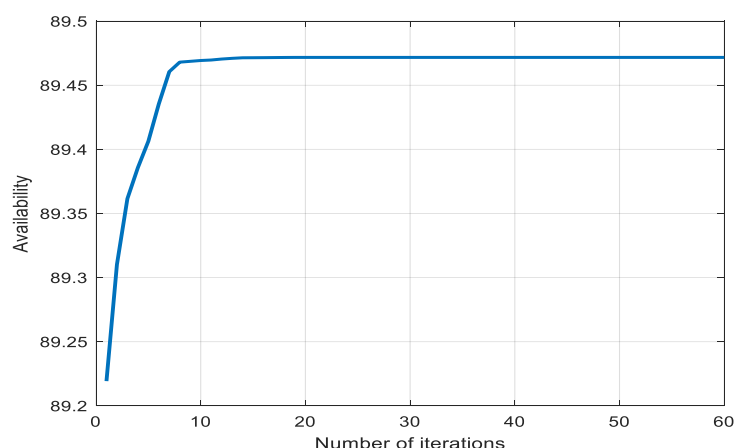


Figure 9. Effect of Number of Iteration on availability of Stock Preparation Unit at population size 1000.

Table 17. Best fitted parameter values of various failure and repair rates of Stock Preparation Unit in a paper plant at different population with different iterations by PSO

Pop.\ Iter		50	60	70	80	90
500	μ_1	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_2	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_3	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_4	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_5	0.0600	0.0600	0.0600	0.0600	0.0600
	θ_1	0.5000	0.5000	0.5000	0.5000	0.5000
	θ_2	0.0800	0.0800	0.0800	0.0800	0.0800
	θ_3	0.9000	0.9000	0.9000	0.9000	0.9000
	θ_4	0.0600	0.0403	0.0736	0.0850	0.0400
	θ_5	0.3875	0.2787	0.3417	0.2883	0.2537
1000	μ_1	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_2	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_3	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_4	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_5	0.0600	0.0600	0.0600	0.0600	0.0600
	θ_1	0.5000	0.5000	0.5000	0.5000	0.5000
	θ_2	0.0800	0.0800	0.0800	0.0800	0.0800
	θ_3	0.9000	0.9000	0.9000	0.9000	0.9000
	θ_4	0.0536	0.0774	0.0606	0.0704	0.0761
	θ_5	0.1570	0.3415	0.4168	0.3269	0.2195
1500	μ_1	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_2	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_3	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_4	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_5	0.0600	0.0600	0.0600	0.0600	0.0600
	θ_1	0.5000	0.5000	0.5000	0.5000	0.5000
	θ_2	0.0800	0.0800	0.0800	0.0800	0.0800
	θ_3	0.9000	0.9000	0.9000	0.9000	0.9000
	θ_4	0.0487	0.0293	0.0782	0.0346	0.0362

2000	θ_5	0.3599	0.4712	0.3694	0.4090	0.3482
	μ_1	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_2	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_3	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_4	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_5	0.0600	0.0600	0.0600	0.0600	0.0600
	θ_1	0.5000	0.5000	0.5000	0.5000	0.5000
	θ_2	0.0800	0.0800	0.0800	0.0800	0.0800
	θ_3	0.09000	0.09000	0.09000	0.09000	0.09000
	θ_4	0.0652	0.0898	0.0689	0.0796	0.0606
2500	θ_5	0.3516	0.2142	0.3211	0.3207	0.2229
	μ_1	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_2	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_3	0.0200	0.0200	0.0200	0.0200	0.0200
	μ_4	0.0100	0.0100	0.0100	0.0100	0.0100
	μ_5	0.0600	0.0600	0.0600	0.0600	0.0600
	θ_1	0.5000	0.5000	0.5000	0.5000	0.5000
	θ_2	0.0800	0.0800	0.0800	0.0800	0.0800
	θ_3	0.9000	0.9000	0.9000	0.9000	0.9000
	θ_4	0.0396	0.0616	0.0536	0.0339	0.0627
	θ_5	0.3139	0.3625	0.2839	0.3432	0.3798

5. Conclusion:

In present study, reliability analysis and availability prediction of the stock preparation unit in a paper manufacturing plant has been conducted and along with RAMD investigation of all its subsystems. It is observed that system reliability is highly influenced by time and failure rates of subsystems. The subsystem deflaking is the most sensitive and highly influence the performance of the system. It is recommended to operate it with utmost care to enhance the reliability of the whole system. Based on maintainability measures, it is recommended to plan maintenance strategies. The steady state availability of the stock preparation unit is sharply decline with the increase of failure rates and increase with the increase of repair rates. Nature inspired algorithms predict that optimal availability is 0.9207 at a population size of 2500 after 80 iterations using GA that is much higher than the maximum value 0.8947 predicted by PSO at population size of 1000 after 60 iterations. It is observed that GA outperformed PSO in the availability prediction of stock preparation unit. The present investigation performed under various assumptions that can be treated as the limitations of the study and in further work it can be extended. The same methodology may be applied in production industries of same kind. Finally, it is concluded that to enhance the performance of stock preparation unit of paper plant special attention should be given to deflaking unit by adopting proper maintenance and redundancy strategies. The system designers may use estimated parameters values to design highly reliable systems.

Conflicts of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization: RASOOL, S.; MAAN, V. S.; **Data curation:** RASOOL, S.; MAAN, V. S.; **Formal analysis:** RASOOL, S.; KUMAR, A.; **Funding acquisition:** - **Investigation:** RASSOL, S.; MAAN, V. S.; **Methodology:** RASOOL, S.; KUMAR, A.; **Project administration:** SAINI, M.; **Software:** RASOOL, S.; MAAN, V. S.; **Resources:** RASOOL, S.; **Supervision:** SAINI, M.; KUMAR, A.; **Validation:** RASOOL, S.; MAAN, V. S.; **Visualization:** KUMAR, A.; MAAN, V. S.; **Writing - original draft:** RASOOL, S.; MAAN, V. S.; **Writing - review and editing:** SAINI, M.; KUMAR, A.

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